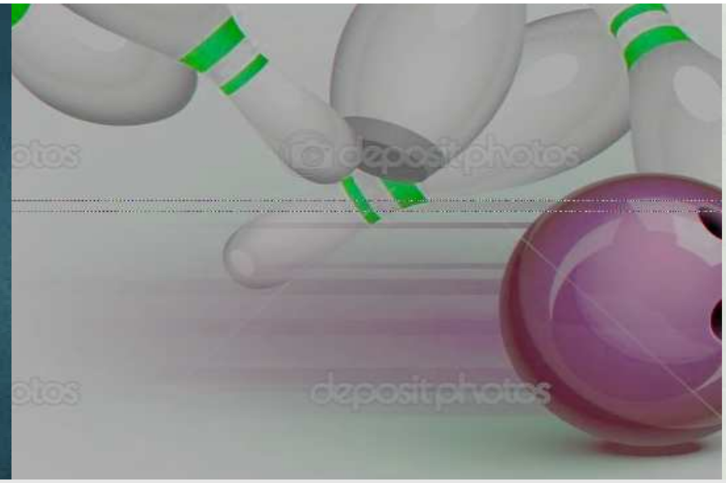
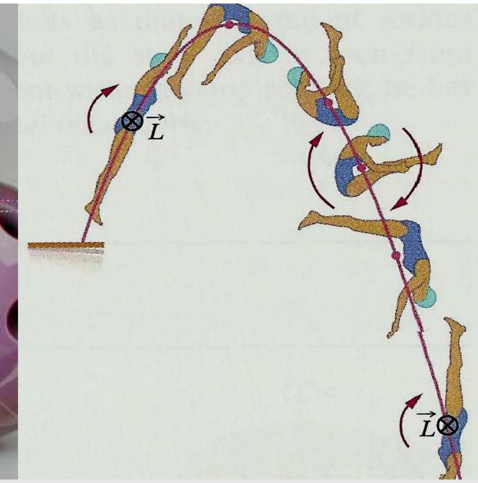





Chapter 11: Rolling, Torque, Angular Momentum


11.2 Rolling as Translational and Rotation Combined

Motion of Translation : i.e.motion along a straight line
Motion of Rotation : rotation about a fixed axis

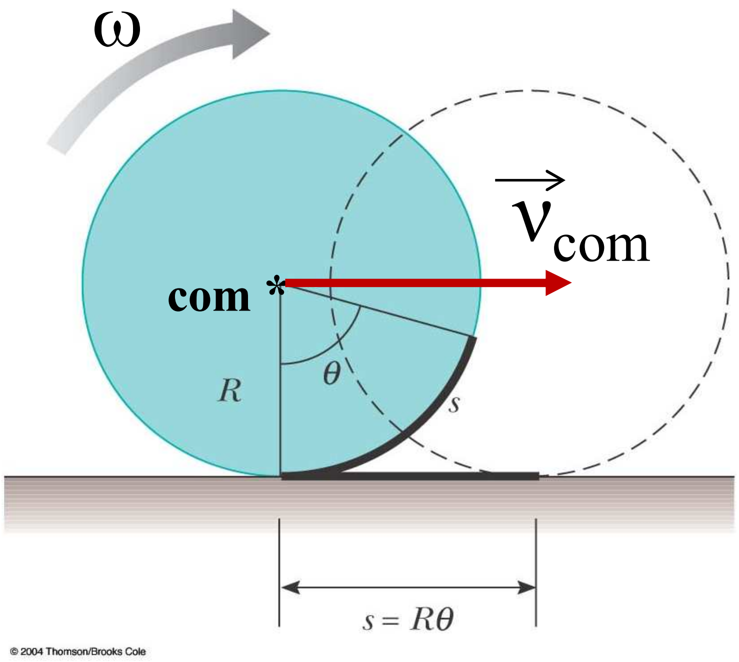
Pure Translation Motion + Pure Rotation Motion = **ROLLING MOTION**

Translation of COM Rotation around COM

Smooth Rolling of a Disk

Fig. 11-2 A time-exposure photograph of a rolling disk. Small lights have been attached to the disk, one at its center and one at its edge. The latter traces out a curve called a *cycloid*. *(Richard Megna/Fundamental Photographs)*

11.2 Rolling

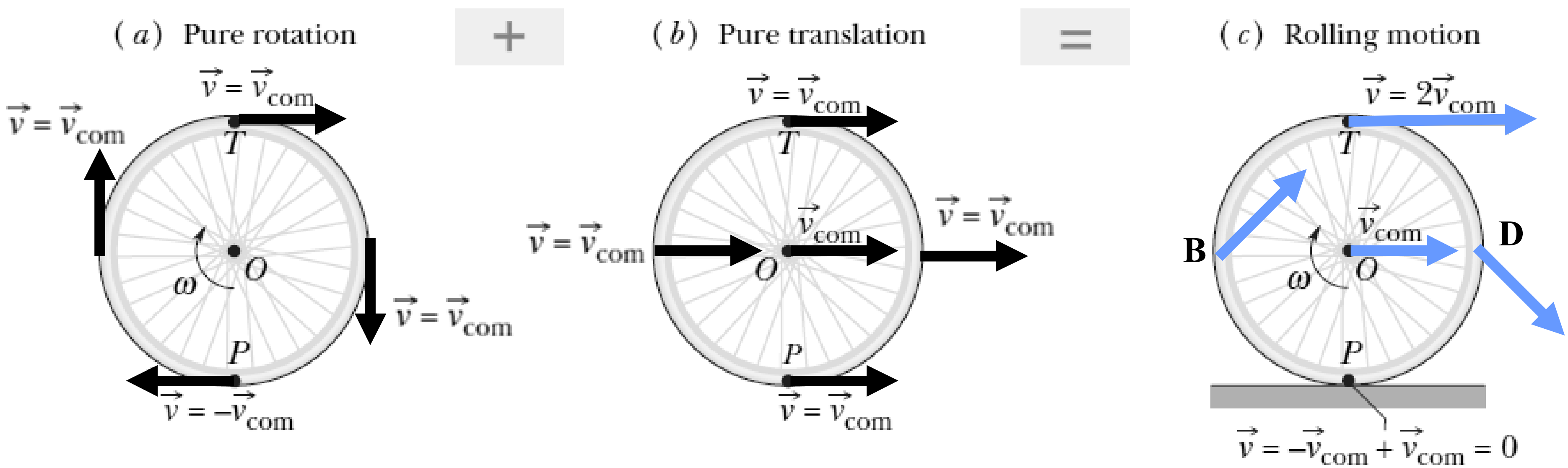


Wheel moves through the arc length of ''s''

$$s = R\theta \Rightarrow \frac{ds}{dt} = \frac{d\theta}{dt}R \Rightarrow v_{com} = \omega R$$

$$a_{com} = \frac{dv_{com}}{dt} = \frac{d\omega}{dt}R = \alpha R$$

11.2 Rolling



Every point on the wheel rotates about com with angular speed of ω

Everypoint outmost part of the wheel has linear speed of v;

Where $\vec{v} = \omega R = \vec{v}_{com}$

All points on the wheel move to the right with same linear velocity, $\vec{v}_{com}$

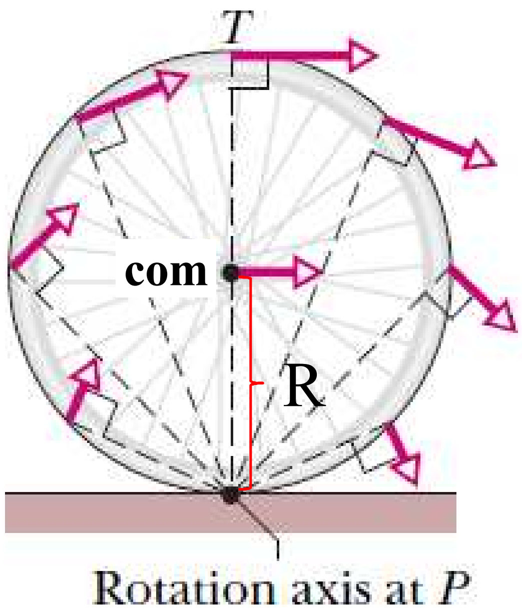
At the bottom of the wheel (point P), the portion of the wheel is stationary

The portion at the top (point T) moving at a speed $2v_{com}$

At points B and D, the speeds are smaller than the point T.

11.2 Rolling

- Rolling can be considered as pure rotation around contact point P. ($V_p = 0$)



($V_p = 0$)

Rotational inertia about com: $\mathbf{I_{com}}$

Rotational inertia about point P:

$$\mathbf{I_P = I_{com} + MR^2}$$

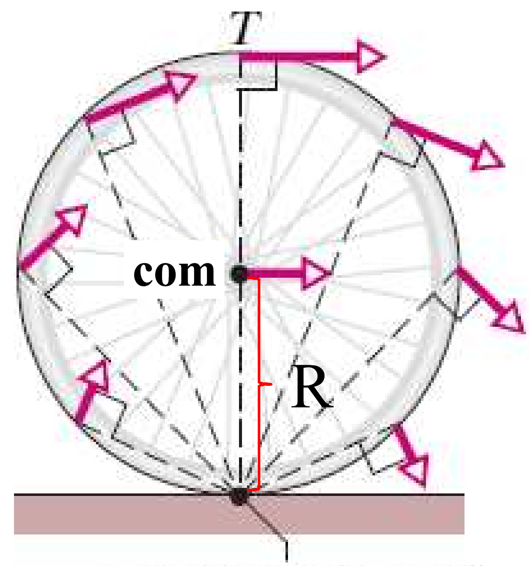
M : mass of the wheel,
I_{com} : rotational inertia about an axis through its center of mass
R : the wheel’s radius, at a perpendicular distance h).

11.3 The Kinetic Energy of Rolling

If we view the rolling as pure rotation about an axis through P, then;

$$K = \tfrac{1}{2} I_P \omega^2$$

ω: angular speed of the wheel
I_P : rotational inertia of the wheel about the axis



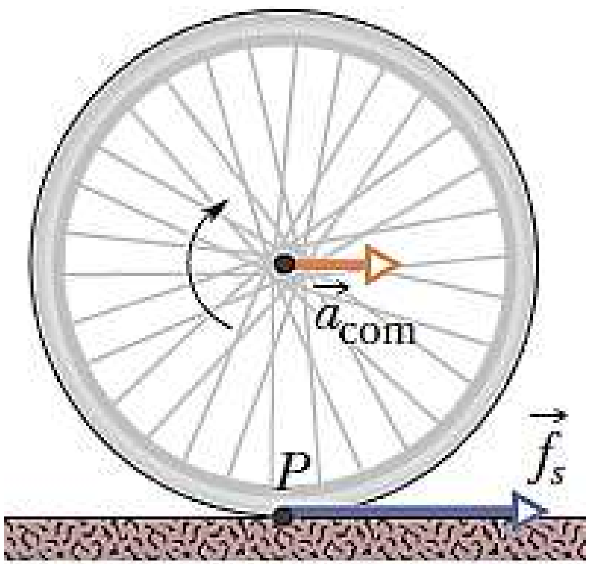
- $I_P = I_{com} + MR^2 \Rightarrow K = ½\, I_{com}\, \omega^2 + ½\, M\, R^2\, \omega^2$
- inserting v_{com} ($v_{com} = \omega R$), then;

$$K = \tfrac{1}{2} I_{com} \omega^2 + \tfrac{1}{2} M v_{com}^2.$$

K.E of ROLLING: Rotational kinetic energy + Translational kinetic energy

11.4: The Forces of Rolling: Friction and Rolling

A wheel rolls horizontally without sliding while accelerating with linear acceleration a_{com}. A static frictional force f_s acts on the wheel at P, opposing its tendency to slide.



The magnitudes of the linear acceleration a_{com}, and the angular acceleration α can be related by;

$$v_{com} = \omega R \Rightarrow \quad a_{com} = \frac{dv_{com}}{dt} = \frac{d\omega}{dt}R = \alpha R$$

$$a_{com} = \alpha R \qquad \text{(smooth rolling motion)}$$

R :the radius of the wheel.

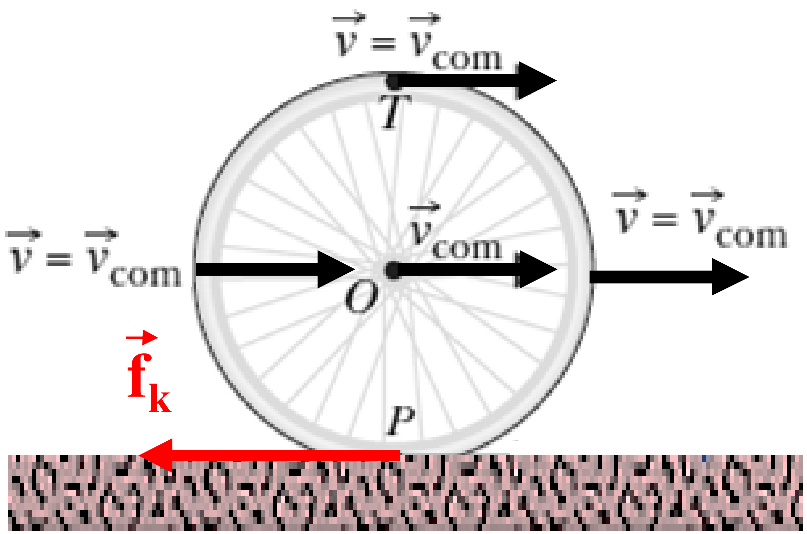
α : angular acceleration

- Rolling is possible when there is friction between the surface and the rolling object.
- The frictional force provides the torque to rotate the object.

11.4: The Forces of Rolling: Friction and Rolling

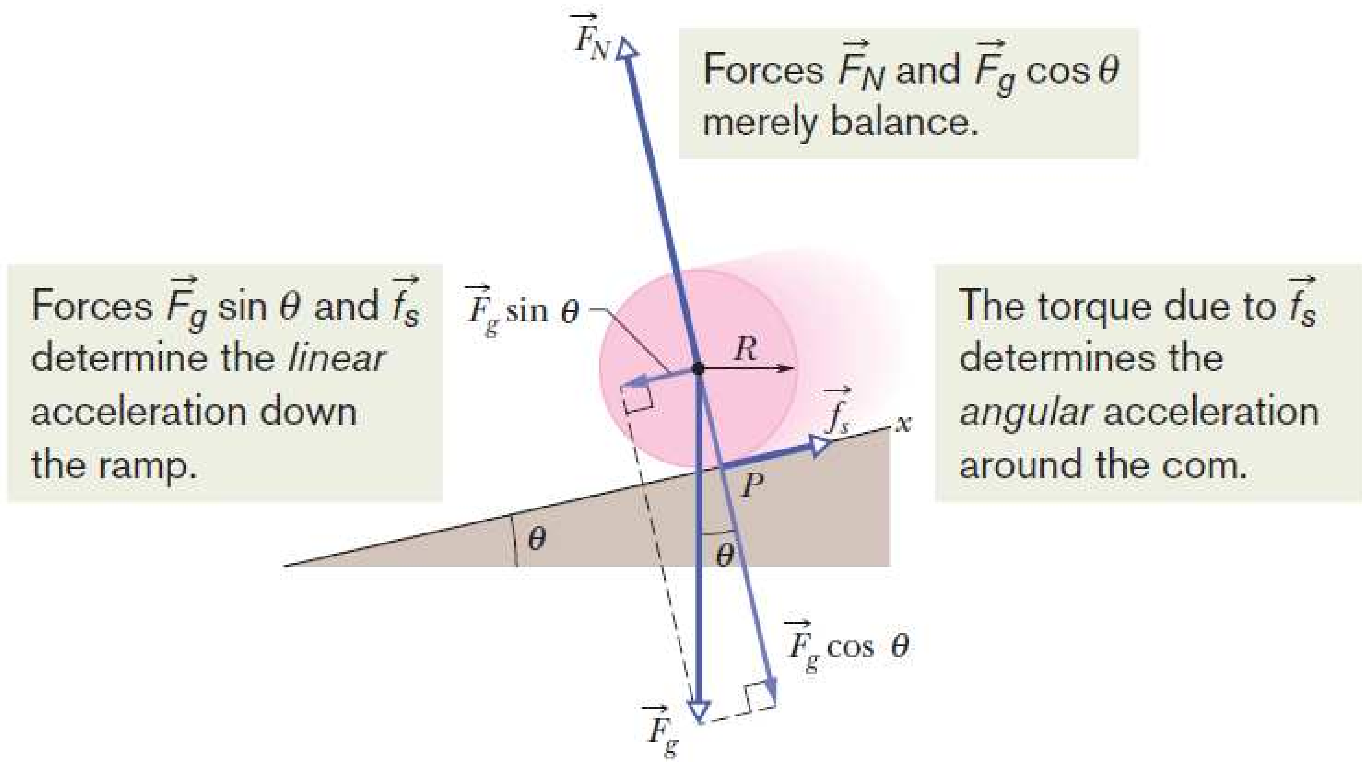
If the wheel slides when the net force acts on it, the frictional force that acts at P in Fig. is a kinetic frictional force, f_k .The motion then is not smooth rolling, and the above relation does not apply to the motion.

Sliding of wheel ⟶ Motion is not smooth rolling



11.4: The Forces of Rolling: Rolling Down a Ramp

A round uniform body of radius R rolls down a ramp. The forces that act on it are the gravitational force F_g, a normal force F_N, and a frictional force f_s pointing up the ramp.

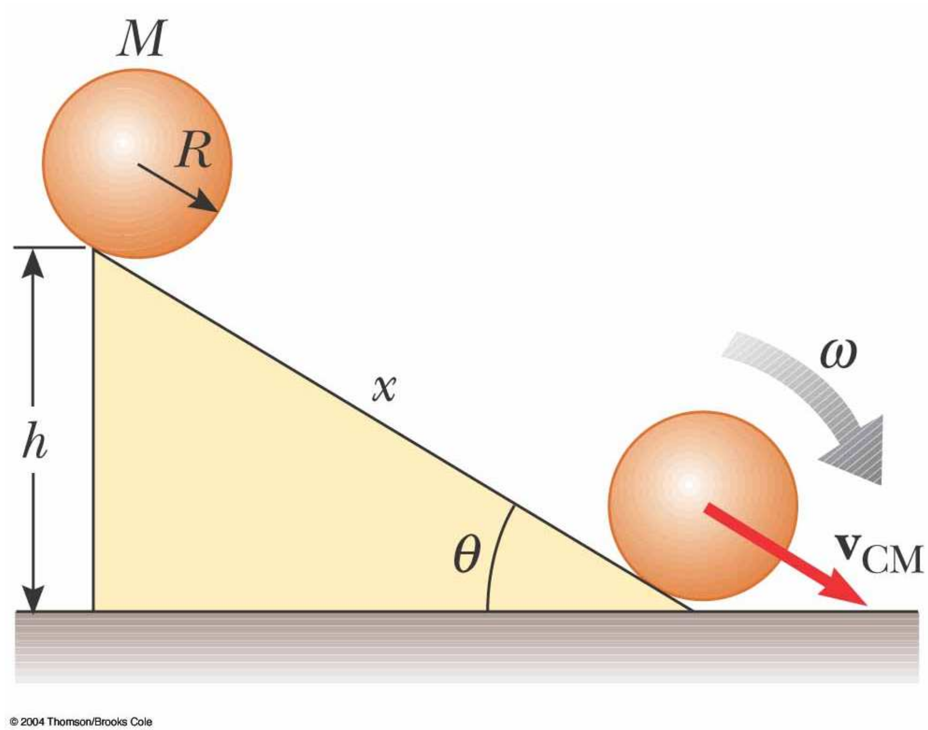


$$\tau_{net} = I\alpha \;\Longrightarrow\; Rf_s = I_{com}\alpha$$

$$a_{com} = \alpha R \;\Longrightarrow\; f_s = -I_{com}\frac{a_{com,x}}{R^2}$$

$$a_{com,x} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$$

11.4: Rolling Down a Ramp



$$K_i + U_i = K_f + U_f$$

$$Mgh = ½\, I_{CM}\, \omega^2 + ½\, M v_{CM}^2$$

$$v_{cm} = \omega R$$

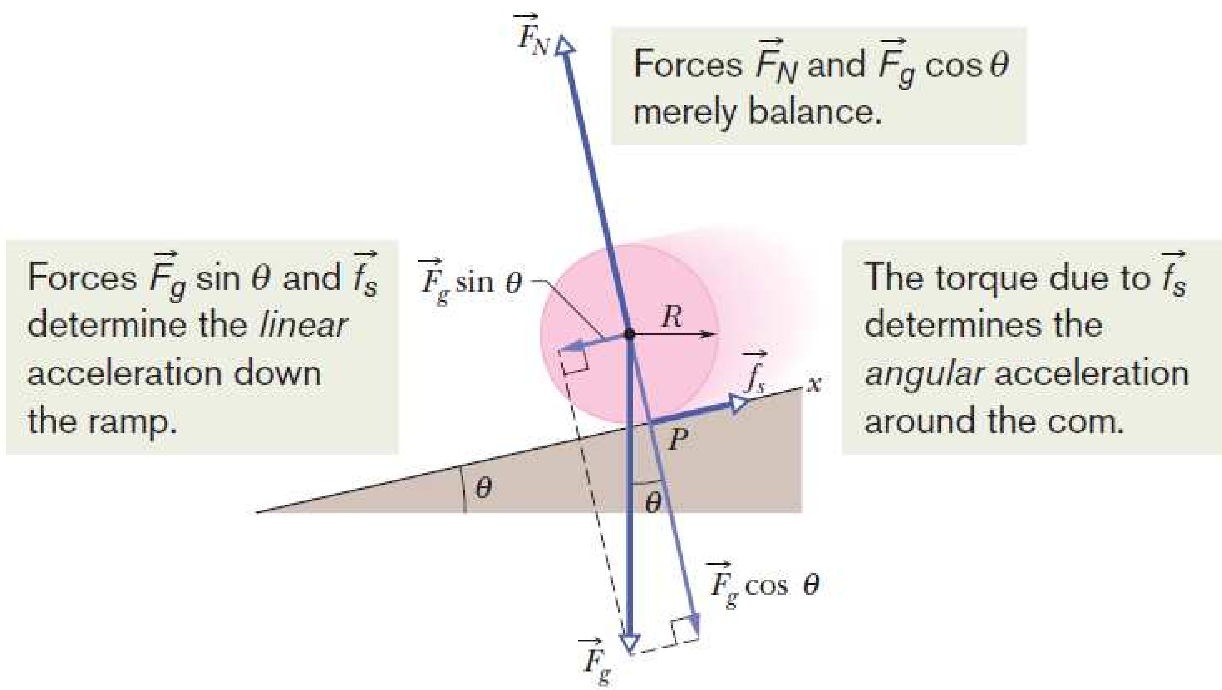
Sample problem:

Rolling Down a Ramp

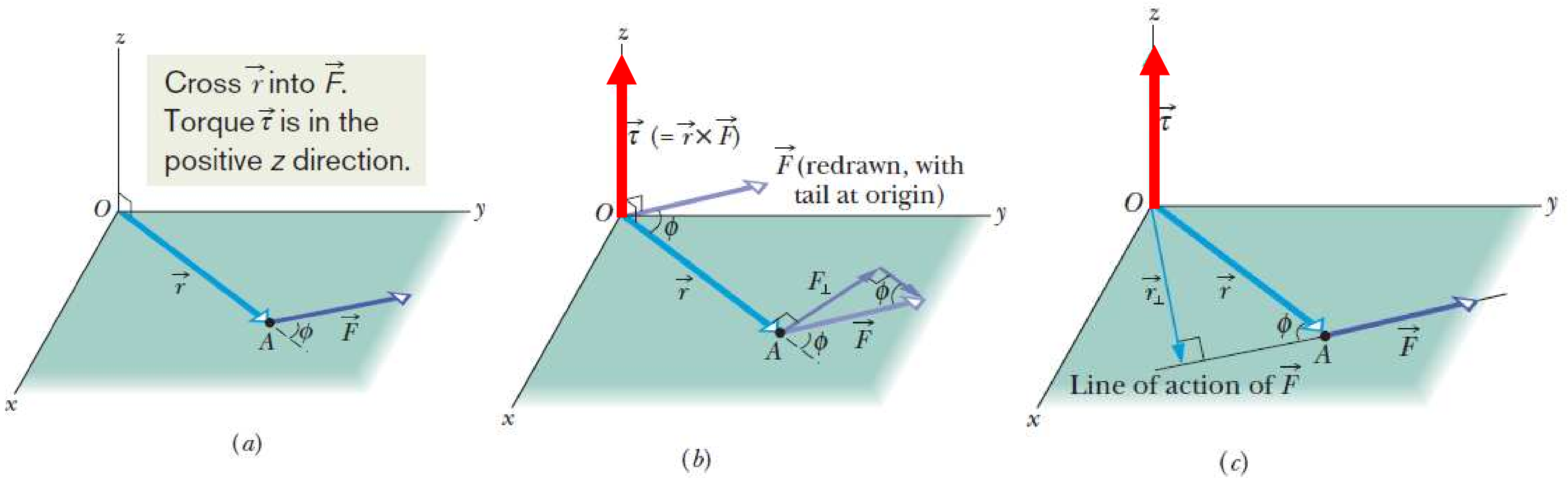
A uniform ball, of mass M=6.00 kg and radius R, rolls smoothly from rest down a ramp at angle θ=30.0°.

a) The ball descends a vertical height h=1.20 m to reach the bottom of the ramp. What is its speed at the bottom?

b) b) What are the magnitude and direction of the frictional force on the ball as it rolls down the ramp?



11.6: Torque Revisited



$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{(torque defined).}$$

$$= rF \sin \phi,$$

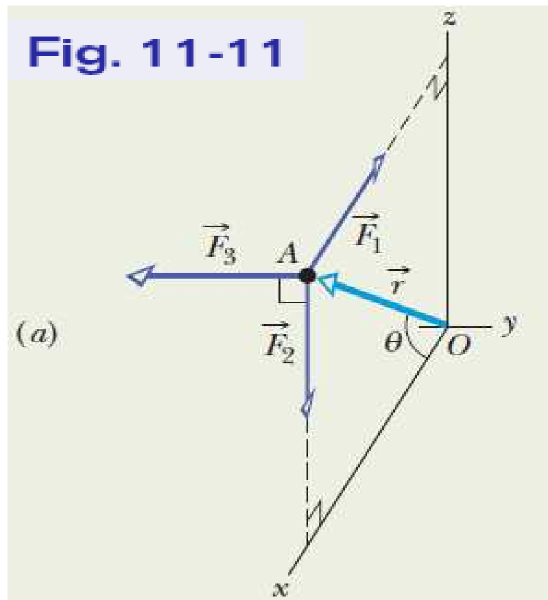
Figure (a) A force F, lying in an x-y plane, acts on a particle at point A. (b) This force produces a torque τ = **r x F** on the particle with respect to the origin O. By the right-hand rule for vector (cross) products, the torque vector points in the positive direction of z. Its magnitude is given by in (b) and by $rF_\perp$ in (c). $r_\perp F$

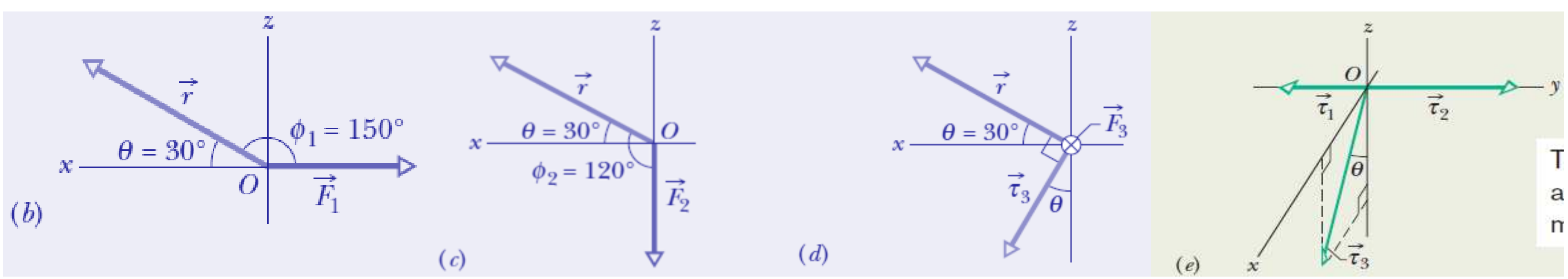
CHECKPOINT 3 The position vector $\vec{r}$ of a particle points along the positive direction of a z axis. If the torque on the particle is (a) zero, (b) in the negative direction of x, and (c) in the negative direction of y, in what direction is the force causing the torque?

Sample problem:

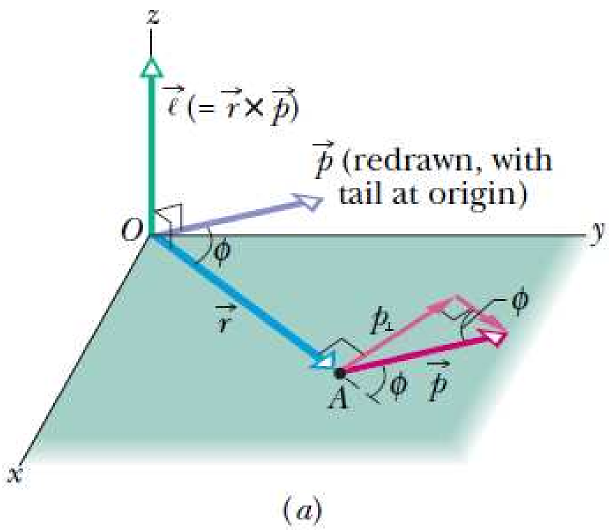
Torque on a particle due to a force

In figure 11-11.a, three forces, each magnitude 2.0 N, act on a particle. The particle is in the xz plane at a point A given by position vector r, where r = 3.0 m and $\theta = 30^o$. Force F is parallel to the x axis, force F_2 is parallel to the z axis, and F_3 is parallel to the y axis. What is the torque, about the origion O, due to each force?





11.7 Angular Momentum



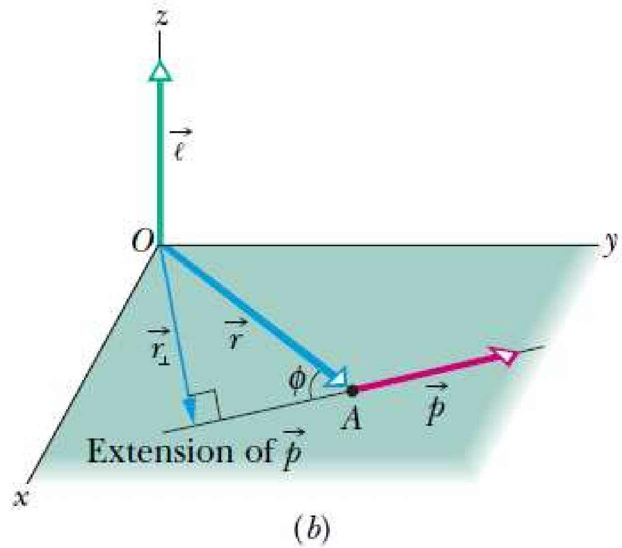


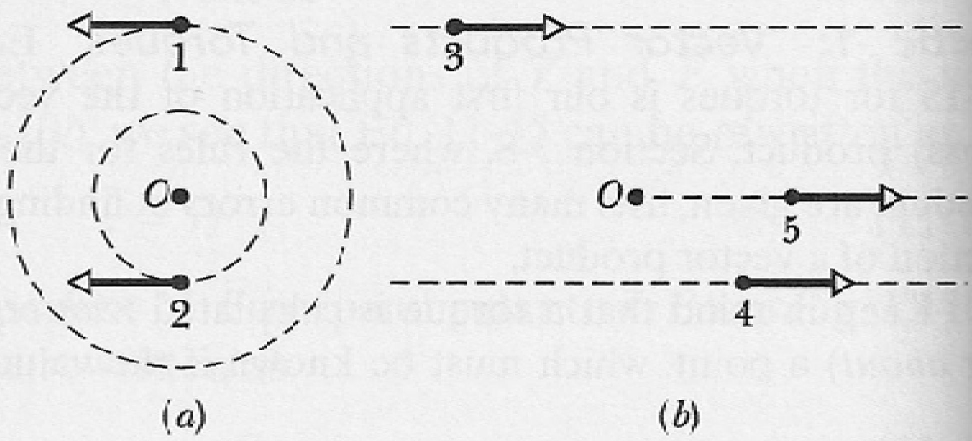
Fig. 11-12 Defining angular momentum. A particle passing through point A has linear momentum $\vec{p}\,(= m\vec{v})$, with the vector $\vec{p}$ lying in an xy plane. The particle has angular momentum $\vec{\ell}\,(= \vec{r} \times \vec{p})$ with respect to the origin O. By the right-hand rule, the angular momentum vector points in the positive direction of z. (a) The magnitude of $\vec{\ell}$ is given by $\ell = rp_\perp = rmv_\perp$. ($b$) The magnitude of $\vec{\ell}$ is also given by $\ell = r_\perp p = r_\perp mv$.

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \qquad \text{(angular momentum defined),}$$

$$\ell = rmv\sin\phi = rp_\perp = rmv_\perp = r_\perp p = r_\perp mv$$

11.7 Angular Momentum

CHECKPOINT 4 In part *a* of the figure, particles 1 and 2 move around point *O* in opposite directions, in circles with radii 2 m and 4 m. In part *b*, particles 3 and 4 travel in the same direction, along straight lines at perpendicular distances of 4 m and 2 m from point *O*. Particle 5 moves directly away from *O*. All five particles have the same mass and the same constant speed. (a) Rank the particles according to the magnitudes of their angular momentum about point *O*, greatest first. (b) Which particles have negative angular momentum about point *O*?



Sample problem:

Angular Momentum

Figure 11-13 shows an overhead view of two particles moving at constant momentum along horizontal paths. Particle 1, with momentum magnitude $p_1 = 5.0\ \text{kg}\cdot\text{m/s}$, has position vector $\vec{r}_1$ and will pass 2.0 m from point O. Particle 2, with momentum magnitude $p_2 = 2.0\ \text{kg}\cdot\text{m/s}$, has position vector $\vec{r}_2$ and will pass 4.0 m from point O. What are the magnitude and direction of the net angular momentum $\vec{L}$ about point O of the two-particle system?

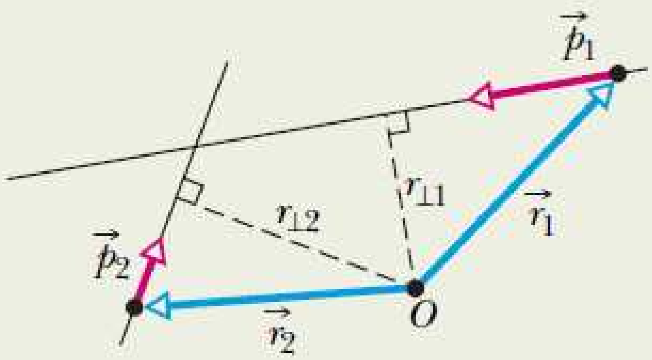


Fig. 11-13 Two particles pass near point O.

11.8: Newton's 2nd Law in Angular Form

Newton's 2nd Law for translation

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}.$$ (Single particle)

Newton's 2nd Law for ANGULAR FORM;

- The (vector) sum of all the torques acting on a particle is equal to the time rate of change of the angular momentum of that particle.

$$\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt}$$ (single particle)

PROOF:

$$\vec{\ell} = m(\vec{r} \times \vec{v}),$$

$$\frac{d\vec{\ell}}{dt} = m\left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v}\right)$$

$$= m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v}).$$

$$= m(\vec{r} \times \vec{a}) = \vec{r} \times m\vec{a}.$$

$$\frac{d\vec{\ell}}{dt} = \underbrace{\vec{r} \times \vec{F}_{net}}_{\vec{\tau}_{net}} = \sum(\vec{r} \times \vec{F}).$$

Sample problem:

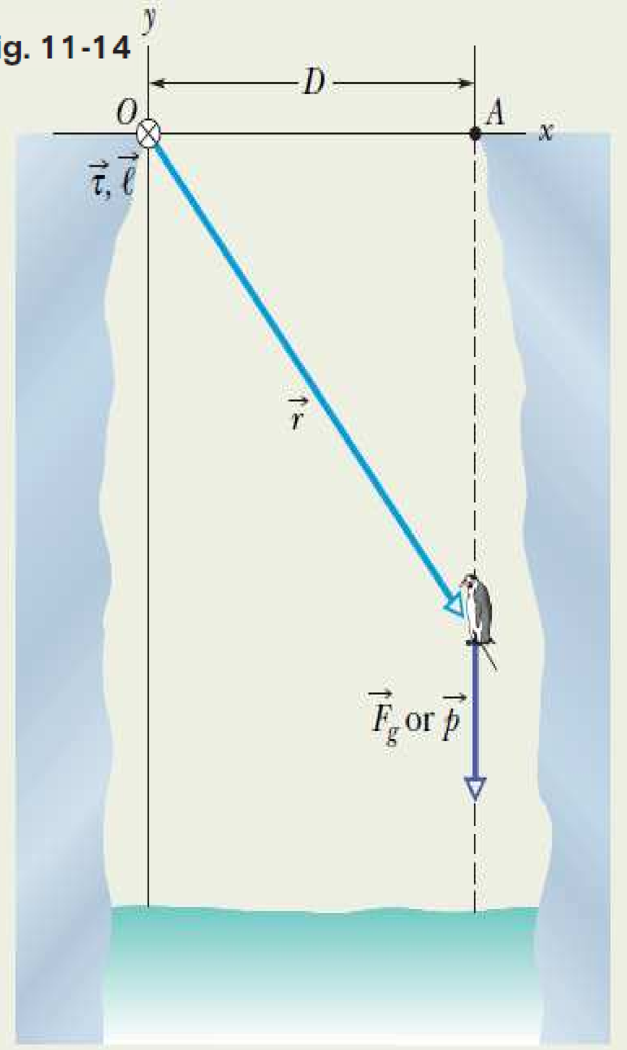
Torque, Penguin Fall

In Fig. 11-14, a penguin of mass m falls from rest at point A, a horizontal distance D from the origin O of an xyz coordinate system. (The positive direction of the z axis is directly outward from the plane of the figure.)

(a) What is the angular momentum $\vec{\ell}$ of the falling penguin about O?

(b) About the origin O, what is the torque on the penguin due to the gravitational force ?

Fig. 11-14



11.9: The Angular Momentum of a System of Particles

The total angular momentum **L** of the system is the (vector) sum of the angular momenta ***l*** of the individual particles (here with label *i):*

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i.$$

With time, the angular momenta of individual particles may change because of interactions between the particles or with the outside.

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{\ell}_i}{dt} = \sum_{i=1}^{n} \vec{\tau}_{\text{net},i}.$$

Therefore, the net external torque acting on a system of particles is equal to the time rate of change of the system's total angular momentum **L**.

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \qquad \text{(system of particles)}$$

11.10: Angular Momentum of a Rigid Body Rotating About a Fixed Axis

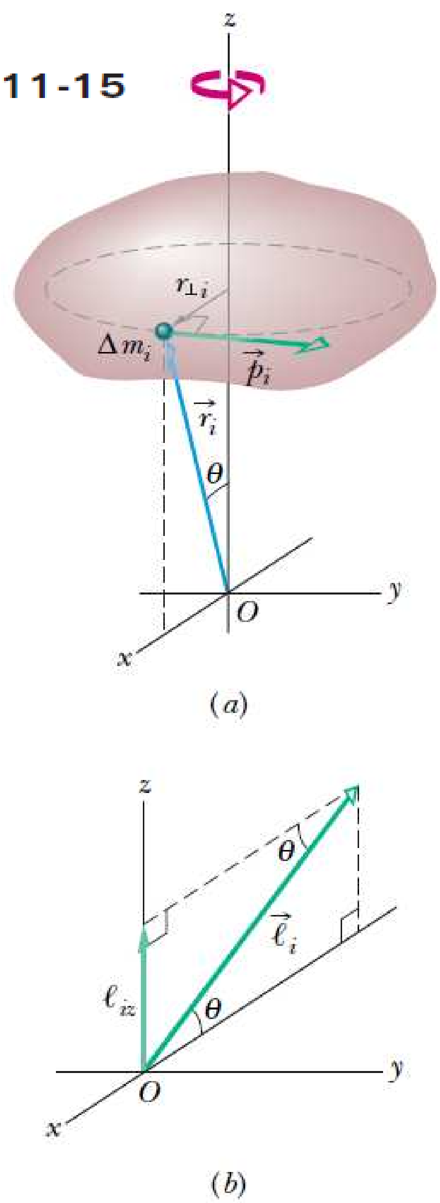


Fig. 11-15

a) A rigid body rotates about a z axis with angular speed ω. A mass element of mass Δm_i within the body moves about the z axis in a circle with radius $r_{\perp i}$. The mass element has linear momentum p_i and it is located relative to the origin O by position vector r_i. Here the mass element is shown when $r_{\perp i}$ is parallel to the x axis.

b) The angular momentum l_i, with respect to O, of the mass element in (a). The z component $\boldsymbol{l}_{iz}$ is also shown.

$$\ell_i = (r_i)(p_i)(\sin 90°) = (r_i)(\Delta m_i\, v_i)$$

$$\ell_{iz} = \ell_i \sin\theta = (r_i \sin\theta)(\Delta m_i\, v_i) = r_{\perp i}\,\Delta m_i\, v_i.$$

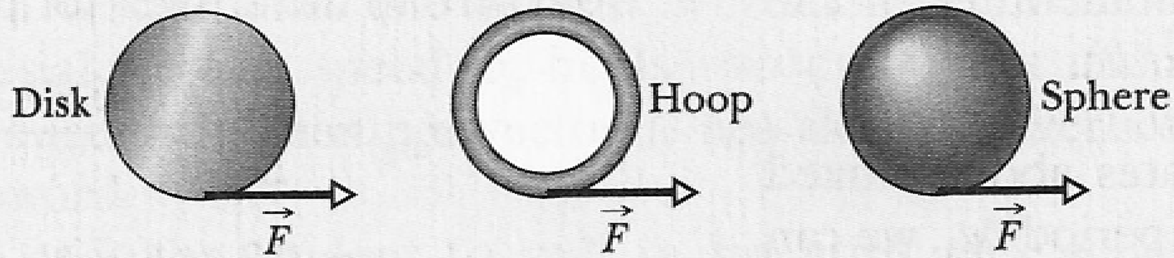
$$L_z = \sum_{i=1}^{n} \ell_{iz} = \sum_{i=1}^{n} \Delta m_i\, v_i r_{\perp i} = \sum_{i=1}^{n} \Delta m_i (\omega r_{\perp i}) r_{\perp i}$$

$$= \omega\left(\sum_{i=1}^{n} \Delta m_i\, r_{\perp i}^2\right).$$

$$L = I\omega \qquad \text{(rigid body, fixed axis).}$$

11.10: Angular Momentum of a Rigid Body Rotating About a Fixed Axis

CHECKPOINT 6 In the figure, a disk, a hoop, and a solid sphere are made to spin about fixed central axes (like a top) by means of strings wrapped around them, with the strings producing the same constant tangential force $\vec{F}$ on all three objects. The three objects have the same mass and radius, and they are initially stationary. Rank the objects according to (a) their angular momentum about their central axes and (b) their angular speed, greatest first, when the strings have been pulled for a certain time t.



11.11: Conservation of Angular Momentum

If the net external torque acting on a system is zero, the angular momentum **L** of the system remains constant, no matter what changes take place within the system.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$$\overrightarrow{\tau_{net}} = 0 \Rightarrow \frac{d\vec{l}}{dt} = 0 \Rightarrow \vec{L} = \text{a constant} \quad \text{(isolated system)}$$

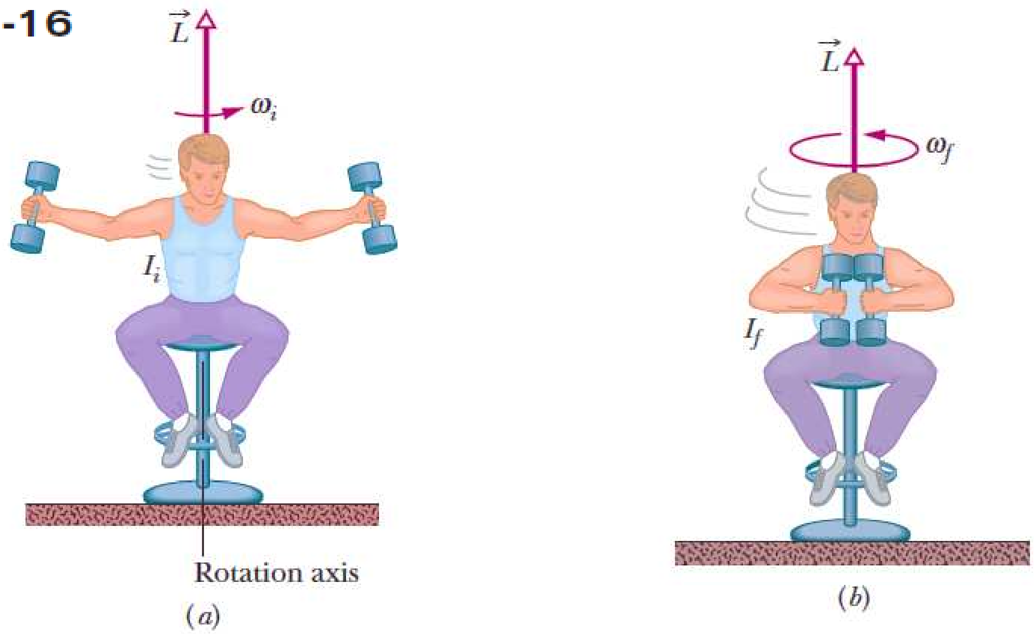
$$\vec{L}_i = \vec{L}_f \quad \text{(isolated system)}$$

- Depending on the torques acting on the system, the angular momentum of the system might be conserved in one or two directions but not in all directions. This means that if external torque along an axis is zero then L is constant.

11.11: Conservation of Angular Momentum

If the component of the net *external* torque on a system along a certain axis is zero, then the component of the angular momentum of the system along that axis cannot change, no matter what changes take place within the system.

Fig. 11-16

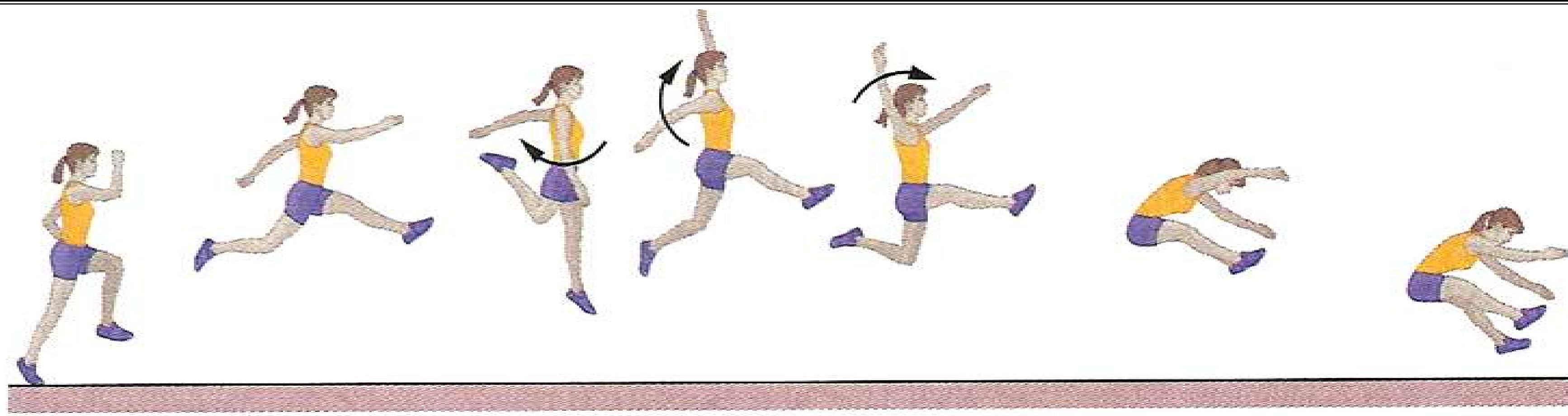


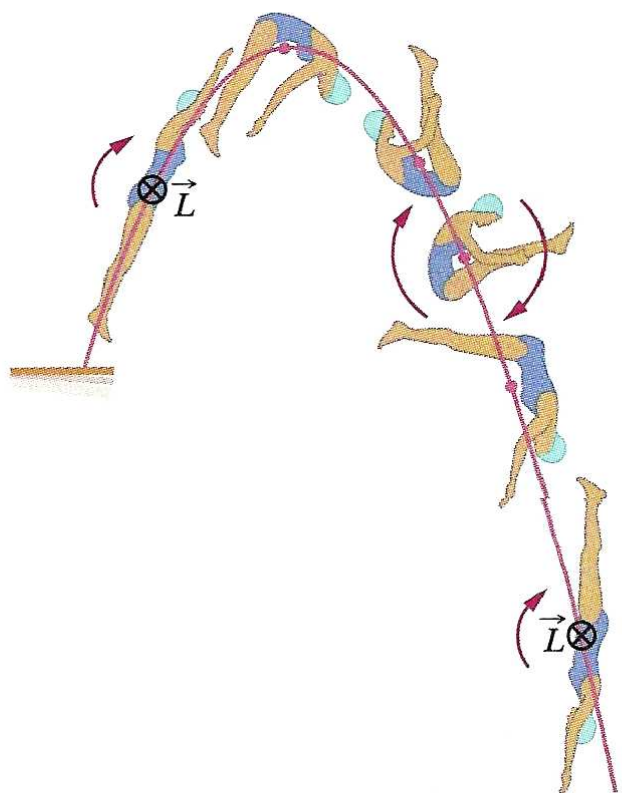
$$I_i\omega_i = I_f\omega_f.$$

(a) The student has a relatively large rotational inertia about the rotation axis and a relatively small angular speed.

(b) By decreasing his rotational inertia, the student automatically increases his angular speed. The angular momentum of the rotating system remains unchanged.

11.11: Conservation of Angular Momentum

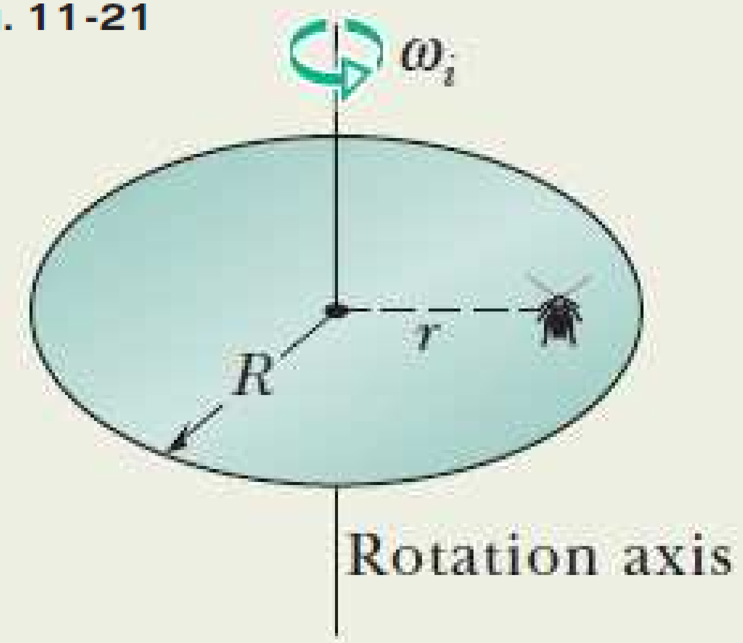




Sample problem

In Fig. 11-21, a cockroach with mass m rides on a disk of mass $6.00m$ and radius R. The disk rotates like a merry-go-round around its central axis at angular speed $\omega_i = 1.50$ rad/s. The cockroach is initially at radius $r = 0.800R$, but then it crawls out to the rim of the disk. Treat the cockroach as a particle. What then is the angular speed?

Fig. 11-21



11.10: Corresponding Variables and Relations for Translational and Rotational Motion[a]

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{\text{com}}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{\text{net}} = \dfrac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{\text{net}} = \dfrac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

[a]See also Table 10-3.
[b]For systems of particles, including rigid bodies.
[c]For a rigid body about a fixed axis, with L being the component along that axis.
[d]For a closed, isolated system.