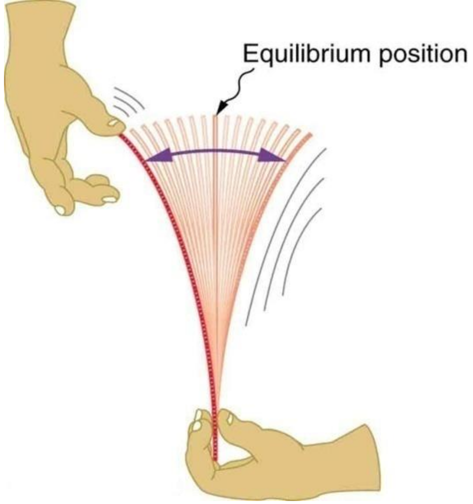
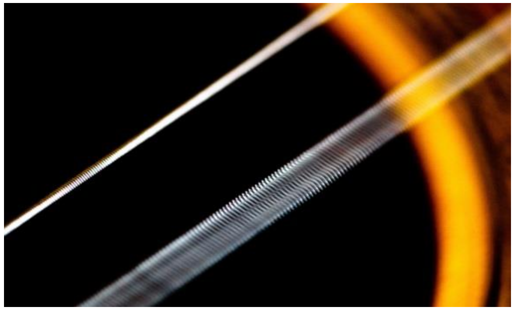
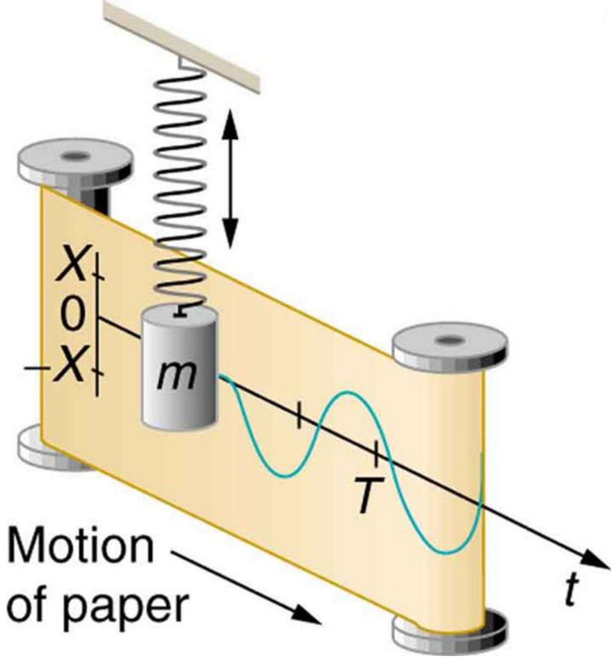
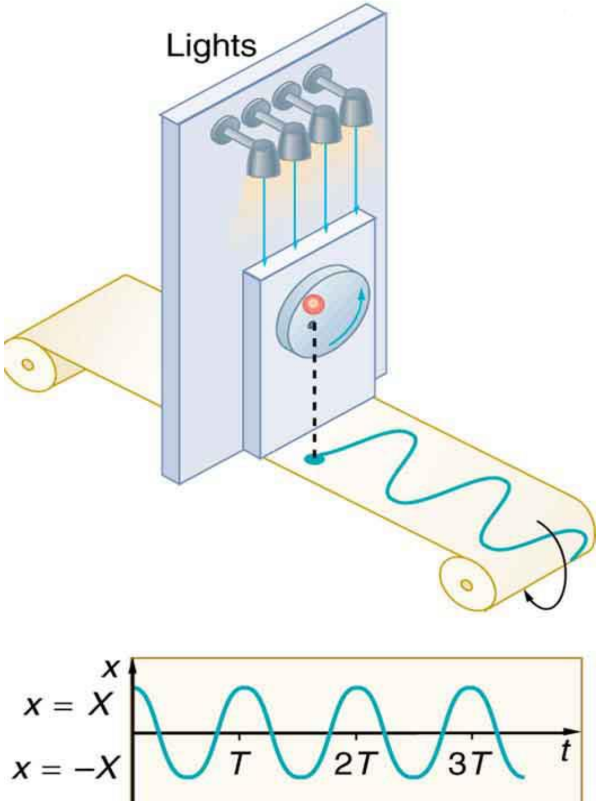






Chapter 15

Oscillations



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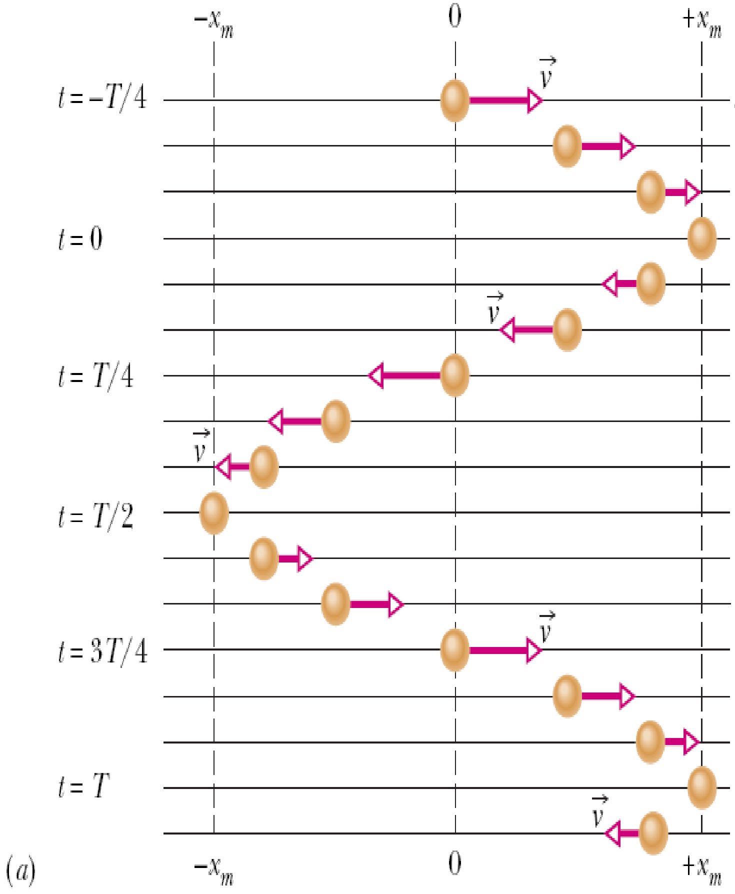
15.2 Simple Harmonic Motion

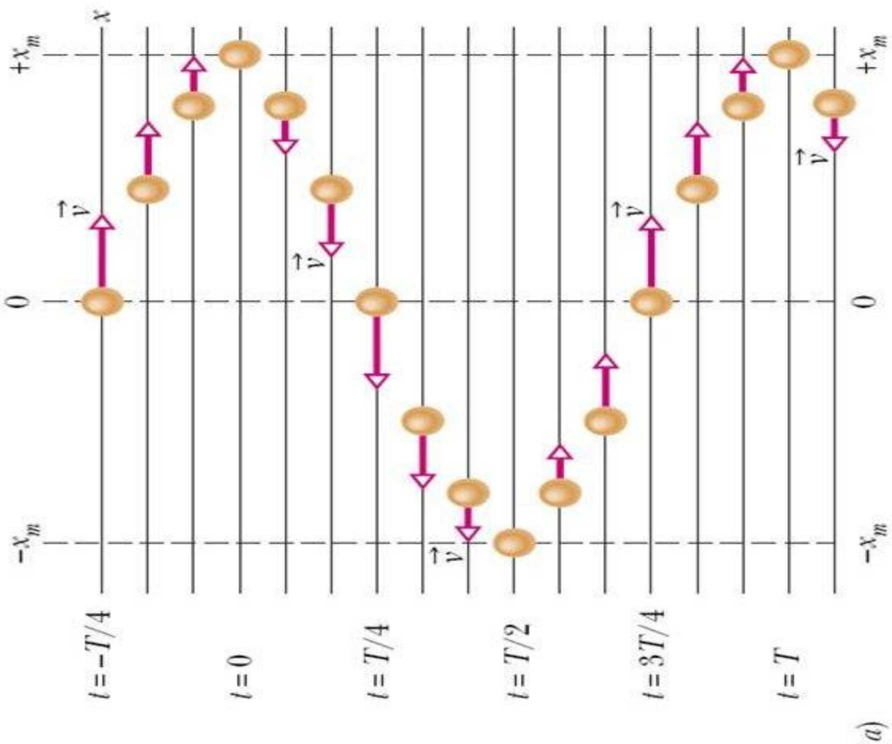
Motion which is periodic in time, that is, motion that repeats itself in time. Harmonic.

In the figure snapshots of a simple oscillatory system is shown. A particle repeatedly moves back and forth about the point x=0.

The time taken for one complete oscillation is the period, T. In the time of one T, the system travels from $x=+x_m$, to $-x_m$, and then back to its original position x_m.

The velocity vector arrows are scaled to indicate the magnitude of the speed of the system at different times. At $x=\pm x_m$, the velocity is zero.





Frequency of oscillation is the number of oscillations that are completed in each second.

The symbol for frequency is f, and the SI unit is the hertz (abbreviated as Hz).

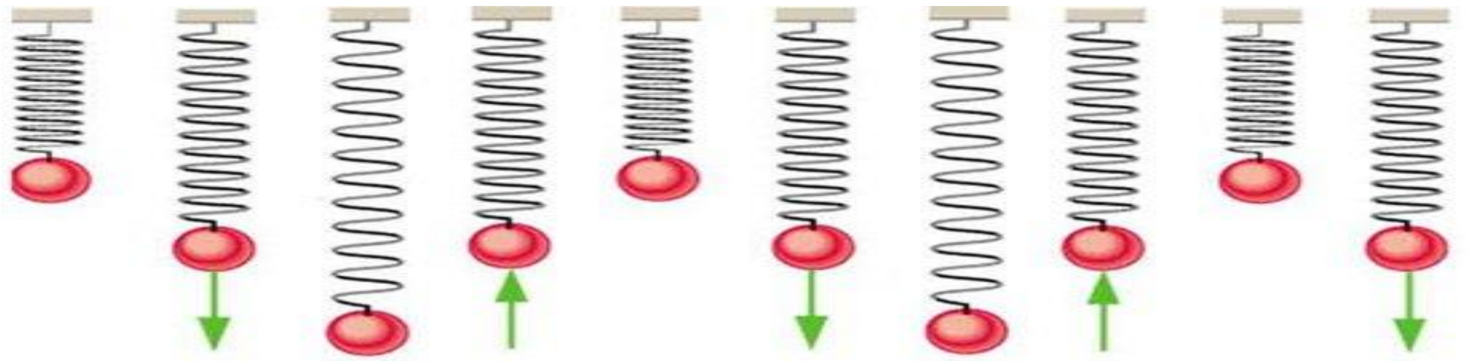
It follows that $$T = \frac{1}{f}$$

If the motion is a sinusoidal function of time, it is called simple harmonic motion (SHM).

$$x(t) = x_m \cos(\omega t + \phi)$$

- x_m is the amplitude (maximum displacement of the system)
- t is the time
- □ ω is the angular frequency, and
- □ ϕ is the phase constant or phase angle

15.2 Simple Harmonic Motion



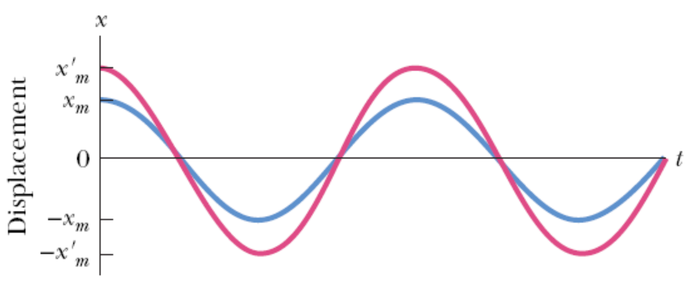
$x = +X$

$x = 0$

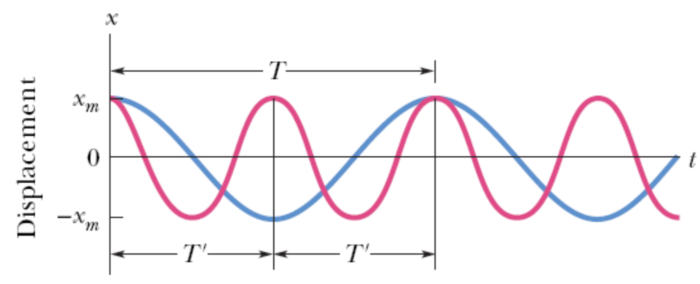
$x = -X$

two SHM systems

different in periods, same amplitude.

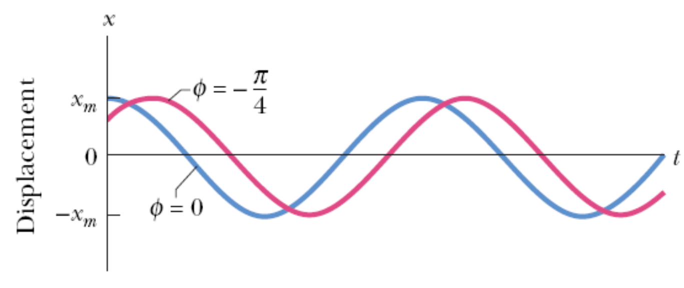


(a)



(b)

different in amplitudes, same period.



(c)

same period and amplitude, different phase constants

The value of the phase constant term, ϕ, depends on the value of the displacement and the velocity of the system at time t = 0.

15.2 Simple Harmonic Motion

For an oscillatory motion with period T,

$$x(t) = x(t+T)$$

The cosine function also repeats itself when the argument increases by 2π. Therefore,

$$\omega(t+T) = \omega t + 2\pi$$

$$\rightarrow \omega T = 2\pi$$

$$\rightarrow \omega = \frac{2\pi}{T} = 2\pi f$$

Here, ω is the angular frequency, and measures the angle per unit time. (ω: radians/second, ϕ: radians)

15.2 Simple Harmonic Motion

- Motion in SHM: $x(t) = x_m \cos(\omega t + \phi)$

- The velocity of SHM: $v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$

$$\rightarrow v(t) = -\omega x_m \sin(\omega t + \phi)$$

The maximum value (amplitude) of velocity is ωx_m. The phase shift of the velocity is $\pi/2$, making the cosine to a sine function.

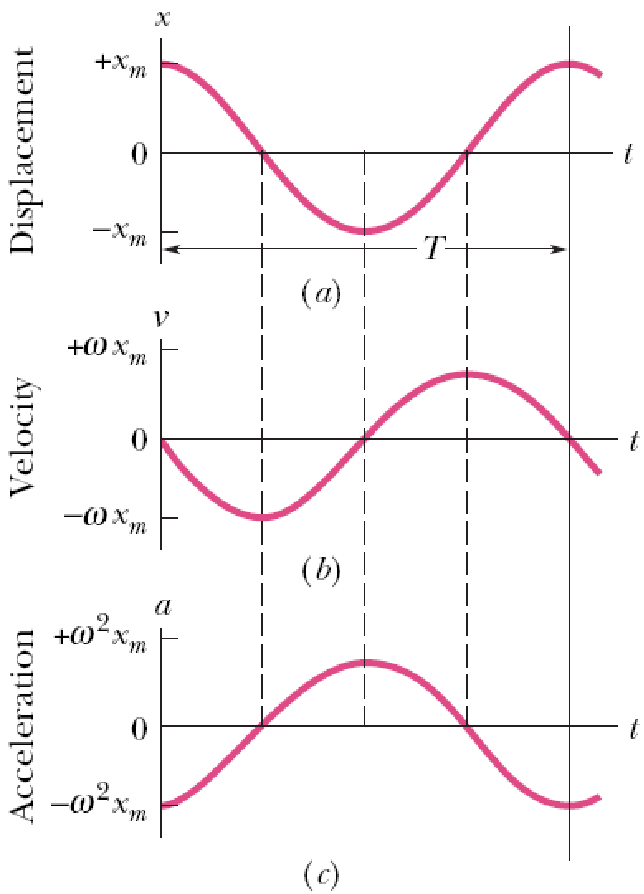
- The acceleration of SHM is:

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]$$

$$\rightarrow a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$\rightarrow a(t) = -\omega^2 x(t)$$

The acceleration amplitude is $\omega^2 x_m$.



In SHM a(t) is proportional to the displacement but opposite in sign.

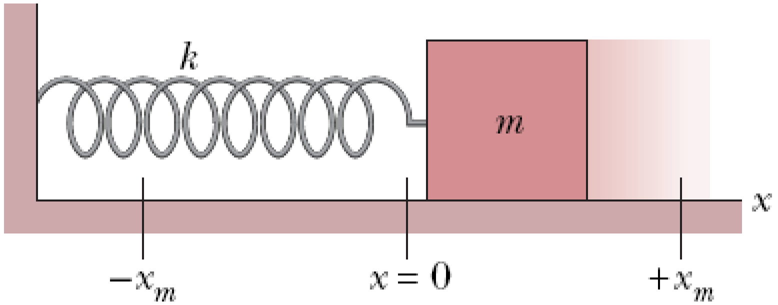
From Newton's 2nd law:

$$F = ma = -(m\omega^2)x = -kx$$

15.3 Force Law for SHM

SHM is the motion executed by a system subject to a force that is proportional to the displacement of the system but opposite in sign.

The block-spring system shown on the right forms a linear SHM oscillator.



The spring constant of the spring, k, is related to the angular frequency, ω, of the

$$\omega = \sqrt{\frac{k}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

Sample Problem

Block–spring SHM, amplitude, acceleration, phase constant

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

(b) What is the amplitude of the oscillation?

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

(d) What is the magnitude a_m of the maximum acceleration of the block?

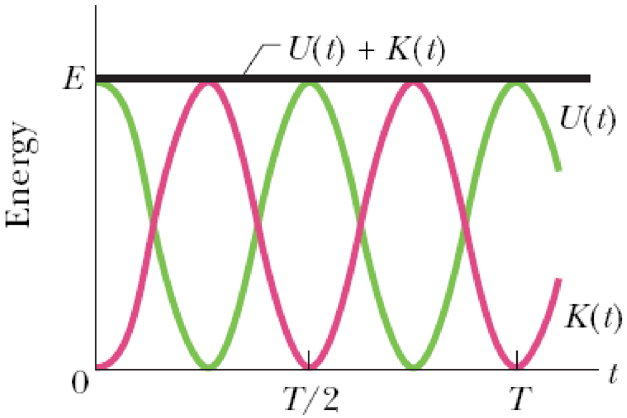
(e) What is the phase constant ϕ for the motion?

(f) What is the displacement function $x(t)$ for the spring–block system?

15.4: Energy in SHM

The potential energy of a linear oscillator is associated entirely with the spring.

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^{\ 2}\cos^2(\omega t + \phi)$$

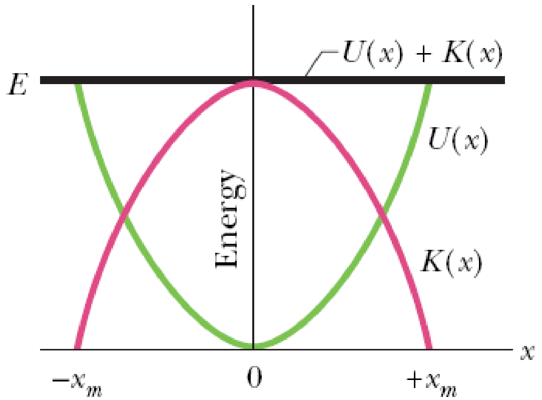


The kinetic energy of the system is associated entirely with the speed of the block.

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_m^{\ 2}\sin^2(\omega t + \phi) = \frac{1}{2}kx_m^{\ 2}\sin^2(\omega t + \phi)$$

The total mechanical energy of the system:

$$E = U + K = \frac{1}{2}kx_m^{\ 2}$$



15.4: Energy in SHM

Sample Problem

SHM potential energy, kinetic energy, mass dampers

Many tall buildings have *mass dampers*, which are anti-sway devices to prevent them from oscillating in a wind. The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say, eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.

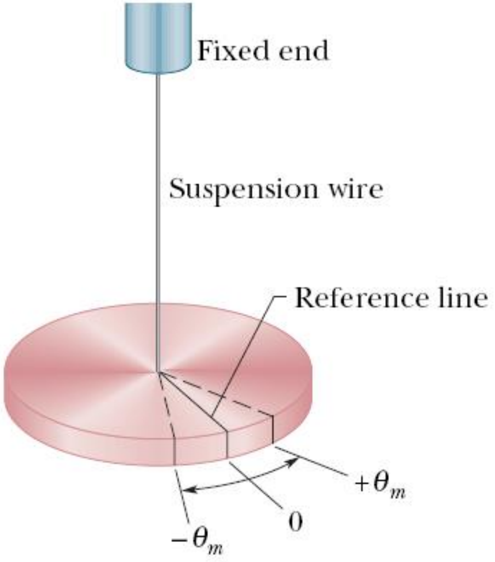
Suppose the block has mass $m = 2.72 \times 10^5$ kg and is designed to oscillate at frequency $f = 10.0$ Hz and with amplitude $x_m = 20.0$ cm.

(a) What is the total mechanical energy E of the spring–block system?

(b) What is the block's speed as it passes through the equilibrium point?

15.4: An Angular SHM

The figure shows an example of angular SHM. In a torsion pendulum involves the twisting of a suspension wire as the disk oscillates in a horizontal plane.



The torque associated with an angular displacement of θ is given by:

$$\tau = -\kappa\theta$$

κ is the torsion constant, and depends on the length, diameter, and material of the suspension wire.

The period, T, relates to κ as:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

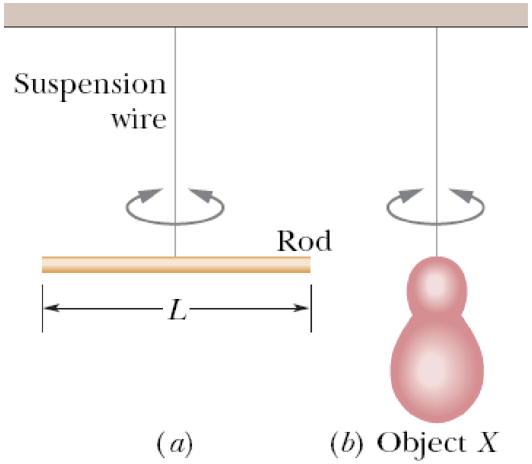
Here, I is the rotational inertia of the oscillating disk.

15.4: An Angular SHM

Sample Problem

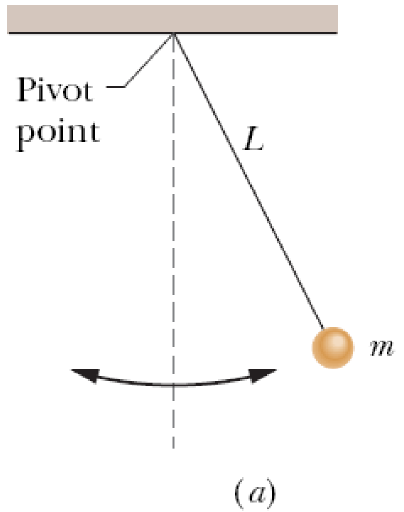
Angular simple harmonic oscillator, rotational inertia, period

Figure 15-8*a* shows a thin rod whose length L is 12.4 cm and whose mass m is 135 g, suspended at its midpoint from a long wire. Its period T_a of angular SHM is measured to be 2.53 s. An irregularly shaped object, which we call object X, is then hung from the same wire, as in Fig. 15-8*b*, and its period T_b is found to be 4.76 s. What is the rotational inertia of object X about its suspension axis?



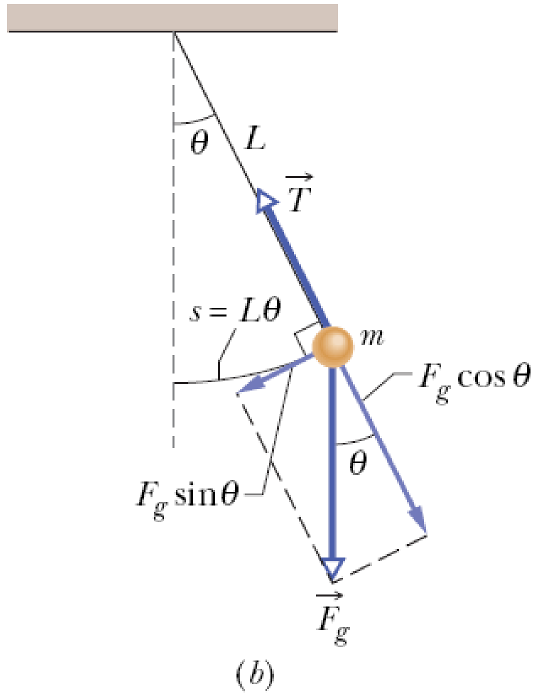
15.6: Pendulums

In a *simple pendulum*, a particle of mass m is suspended from one end of an unstretchable massless string of length L that is fixed at the other end.



The restoring torque acting on the mass when its angular displacement is θ, is:

$$\tau = -L(F_g \sin\theta) = I\alpha$$



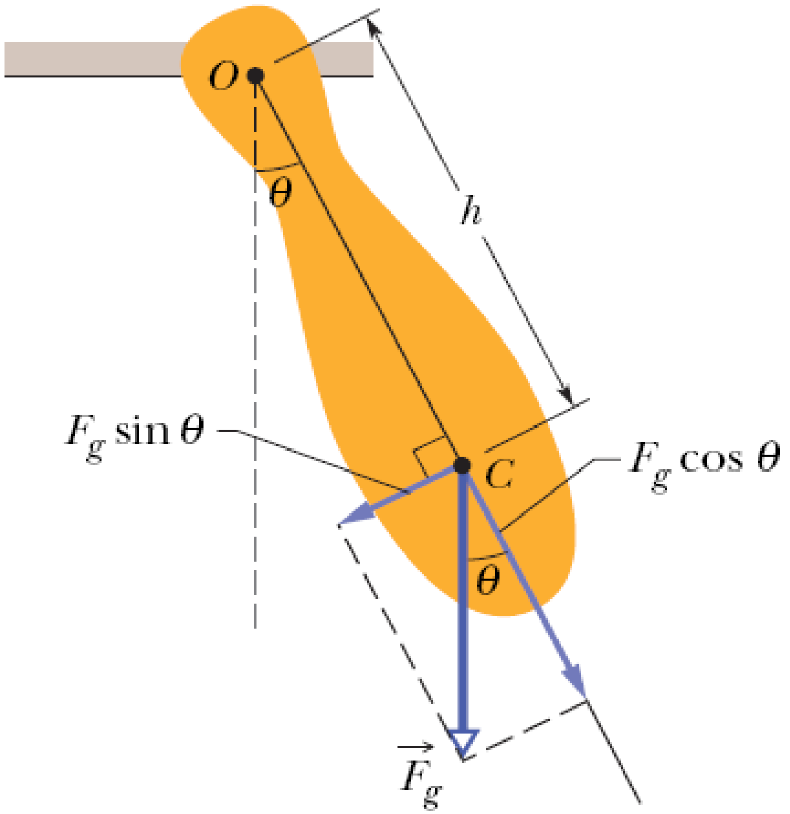
α is the angular acceleration of the mass. Finally,

$$\alpha = -\frac{mgL}{I}\theta, and$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

This is true for *small angular displacements*, θ.

A physical pendulum can have a complicated distribution of mass. If the center of mass, C, is at a distance of h from the pivot point (figure), then for *small angular amplitudes*. the motion is simple harmonic.



The period, T, is:

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Here, I is the rotational inertia of the pendulum about O.

In the **small-angle approximation** we can assume that $\theta << 1$ and use the approximation $\sin\theta \cong \theta$

15.6: Pendulums

Let us investigate up to what angle θ is the approximation reasonably accurate?

θ (degrees)	θ (radians)	$\sin\theta$
5	0.087	0.087
10	0.174	0.174
15	0.262	0.259 (1% off)
20	0.349	0.342 (2% off)

Conclusion: If we keep $\theta < 10°$ we make less than 1 % error.

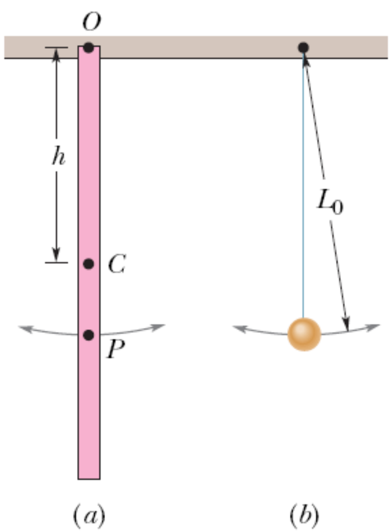
Sample Problem

Physical pendulum, period and length

In Fig. 15-11*a*, a meter stick swings about a pivot point at one end, at distance h from the stick's center of mass.

(a) What is the period of oscillation T?

(b) What is the distance L_0 between the pivot point O of the stick and the center of oscillation of the stick?



15.7: SHM and uniform circular motion

Consider a reference particle P' moving in uniform circular motion with constant angular speed (w).

The projection of the particle on the x-axis is a point P, describing motion given by:

$$x(t) = x_m \cos(\omega t + \phi).$$

This is the displacemnt equation of SHM.

SHM, therefore, is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.