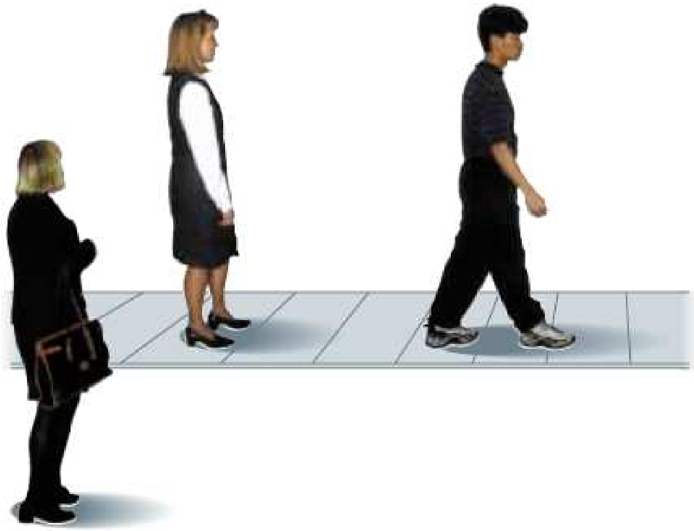
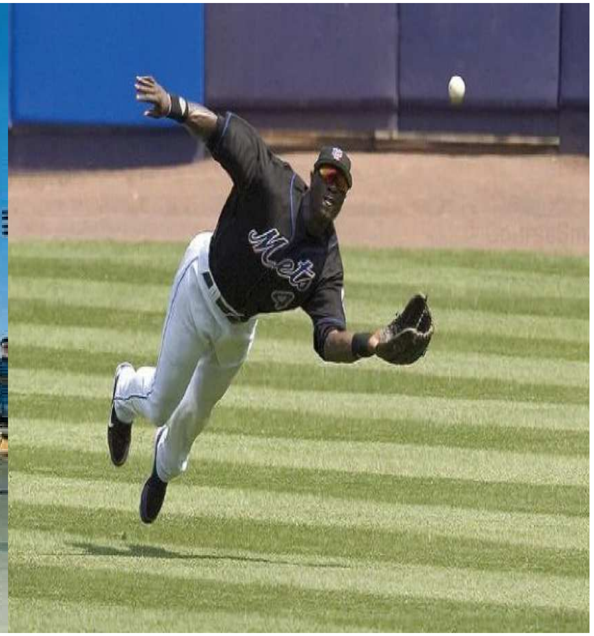


CHAPTER IV

MOTION IN TWO AND THREE DIMENSIONS

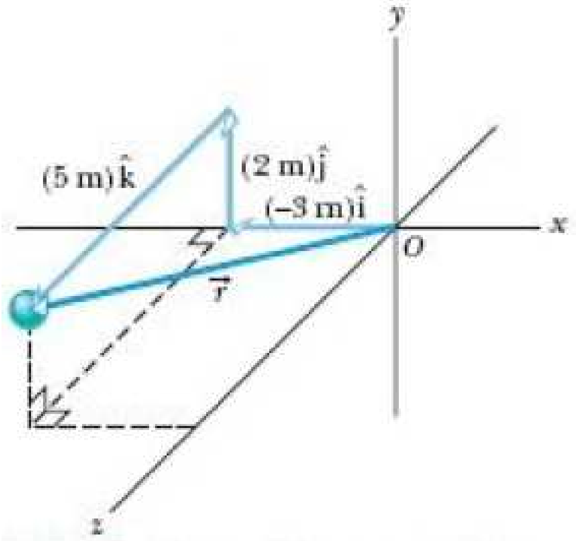


Position and Displacement

Position of a particle

- Described by a position vector, with respect to a reference origin.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



Displacement

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k},$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}.$$

Example 1: Two-dimensional motion (rabbit position)

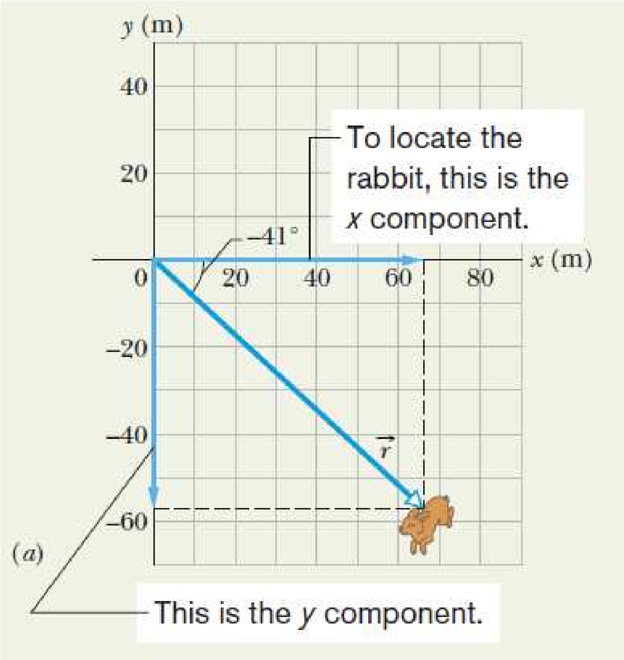
A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \quad (4\text{-}5)$$

and

$$y = 0.22t^2 - 9.1t + 30. \quad (4\text{-}6)$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?



KEY IDEA

The x and y coordinates of the rabbit's position, as given by Eqs. 4-5 and 4-6, are the scalar components of the rabbit's position vector $\vec{r}$.

Calculations: We can write

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}. \quad (4\text{-}7)$$

(We write $\vec{r}(t)$ rather than $\vec{r}$ because the components are functions of t, and thus $\vec{r}$ is also.)

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and

$$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so

$$\vec{r} = (66 \text{ m})\hat{i} - (57 \text{ m})\hat{j}, \quad \text{(Answer)}$$

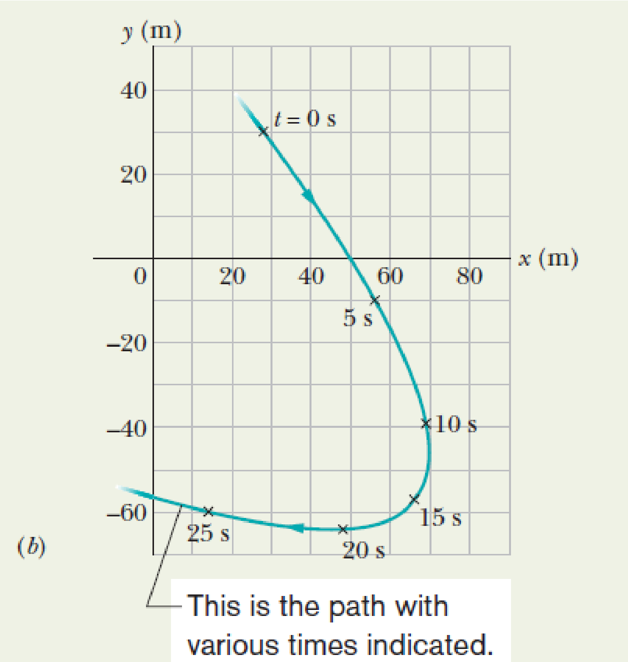
which is drawn in Fig. 4-2a. To get the magnitude and angle of $\vec{r}$, we use Eq. 3-6:

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$

$$= 87 \text{ m}, \quad \text{(Answer)}$$

and

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41°. \quad \text{(Answer)}$$



Average Velocity and Instantaneous Velocity

If a particle moves through a displacement of Δr in Δt time, then the average velocity is:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}},$$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$

In the limit that the Δt time shrinks to a single point in time, the average velocity is approaches instantaneous velocity. This velocity is the derivative of displacement with respect to time.

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

Example 2: Two-dimensional motion (rabbit velocity)

For the rabbit in the preceding Sample Problem, find the velocity $\vec{v}$ at time $t = 15$ s.

KEY IDEA

We can find $\vec{v}$ by taking derivatives of the components of the rabbit's position vector.

Calculations: Applying the v_x part of Eq. 4-12 to Eq. 4-5, we find the x component of $\vec{v}$ to be

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$
$$= -0.62t + 7.2. \qquad (4\text{-}13)$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part of Eq. 4-12 to Eq. 4-6, we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1. \qquad (4\text{-}14)$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation 4-11 then yields

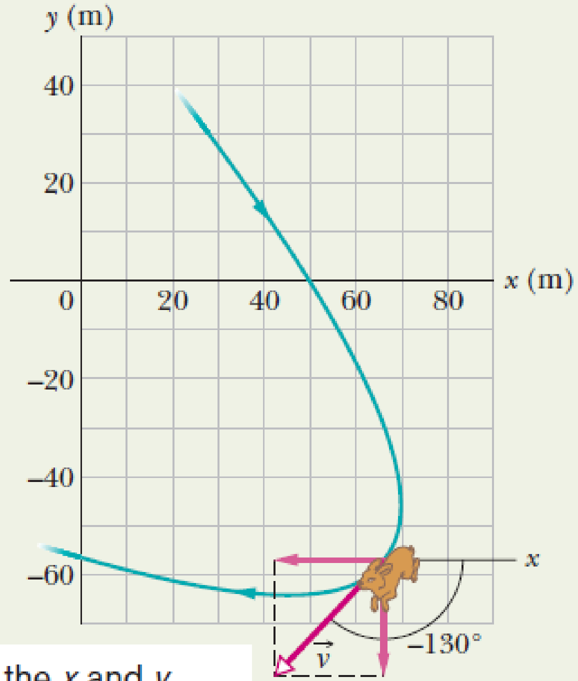
$$\vec{v} = (-2.1 \text{ m/s})\hat{\text{i}} + (-2.5 \text{ m/s})\hat{\text{j}}, \qquad \text{(Answer)}$$

which is shown in Fig. 4-5, tangent to the rabbit's path and in the direction the rabbit is running at $t = 15$ s.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1 \text{ m/s})^2 + (-2.5 \text{ m/s})^2}$$
$$= 3.3 \text{ m/s} \qquad \text{(Answer)}$$

and
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5 \text{ m/s}}{-2.1 \text{ m/s}}\right)$$
$$= \tan^{-1} 1.19 = -130°. \qquad \text{(Answer)}$$

Check: Is the angle $-130°$ or $-130° + 180° = 50°$?



These are the *x* and *y* components of the vector at this instant.

Average and Instantaneous Accelerations

Following the same definition as in average velocity,

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}},$$

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

If we shrink Δt to zero, then the average acceleration value approaches to the instant acceleration value, which is the derivative of velocity with respect to time:

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}})$$

$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}$$

$$= a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}$$

Example 3: Two-dimensional motion (rabbit run)

For the rabbit in the preceding two Sample Problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

KEY IDEA

We can find $\vec{a}$ by taking derivatives of the rabbit's velocity components.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

Similarly, applying the a_y part of Eq. 4-18 to Eq. 4-14 yields the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{i} + (0.44 \text{ m/s}^2)\hat{j}, \quad \text{(Answer)}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62 \text{ m/s}^2)^2 + (0.44 \text{ m/s}^2)^2}$$
$$= 0.76 \text{ m/s}^2. \quad \text{(Answer)}$$

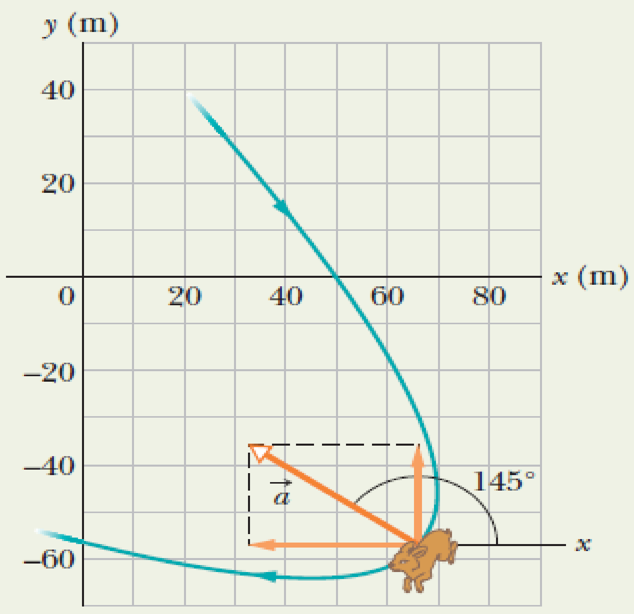
For the angle we have

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2}\right) = -35°.$$

However, this angle, which is the one displayed on a calculator, indicates that $\vec{a}$ is directed to the right and downward in Fig. 4-7. Yet, we know from the components that $\vec{a}$ must be directed to the left and upward. To find the other angle that has the same tangent as $-35°$ but is not displayed on a calculator, we add 180°:

$$-35° + 180° = 145°. \quad \text{(Answer)}$$

This *is* consistent with the components of $\vec{a}$ because it gives a vector that is to the left and upward. Note that $\vec{a}$ has the same magnitude and direction throughout the rabbit's run because the acceleration is constant.



These are the x and y components of the vector at this instant.

Projectile Motion

- Projectile –a particle moves in a vertical plane with some initial velocity but its acceleration is always the free-fall acceleration which is downward.
- This particle's motion is called projectile motion.
- Thrown ball
- Bullet (ballistics considered as projectile motion)
- Dropped package

! IMPORTANT NOTE !

We have assumed that air through which the projectile moves has no effect on its motion.

Examples in sports:

Tennis

Baseball

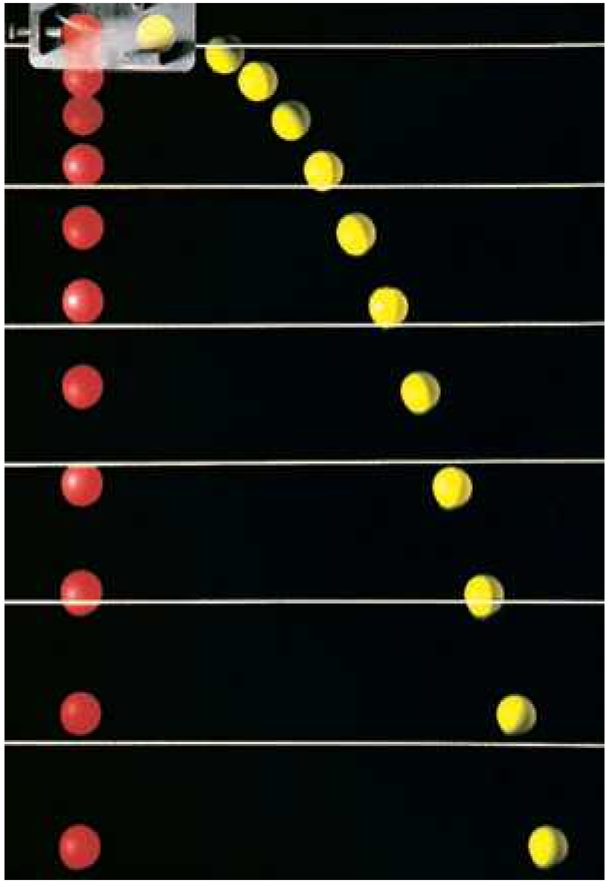
Football

Lacrosse

Racquetball

Soccer…………

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.



A stroboscopic photograph of two golf balls.

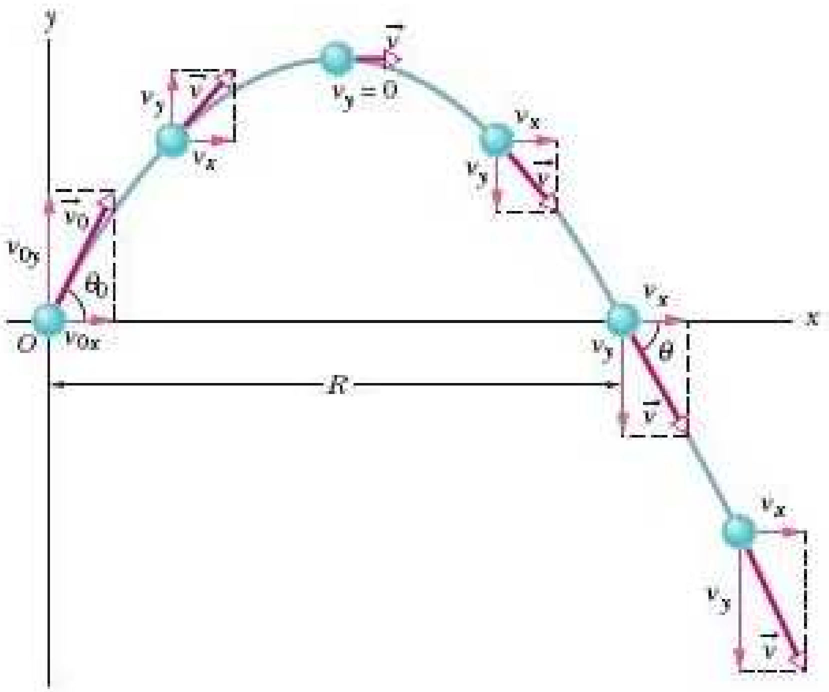
More on Projectile Motion

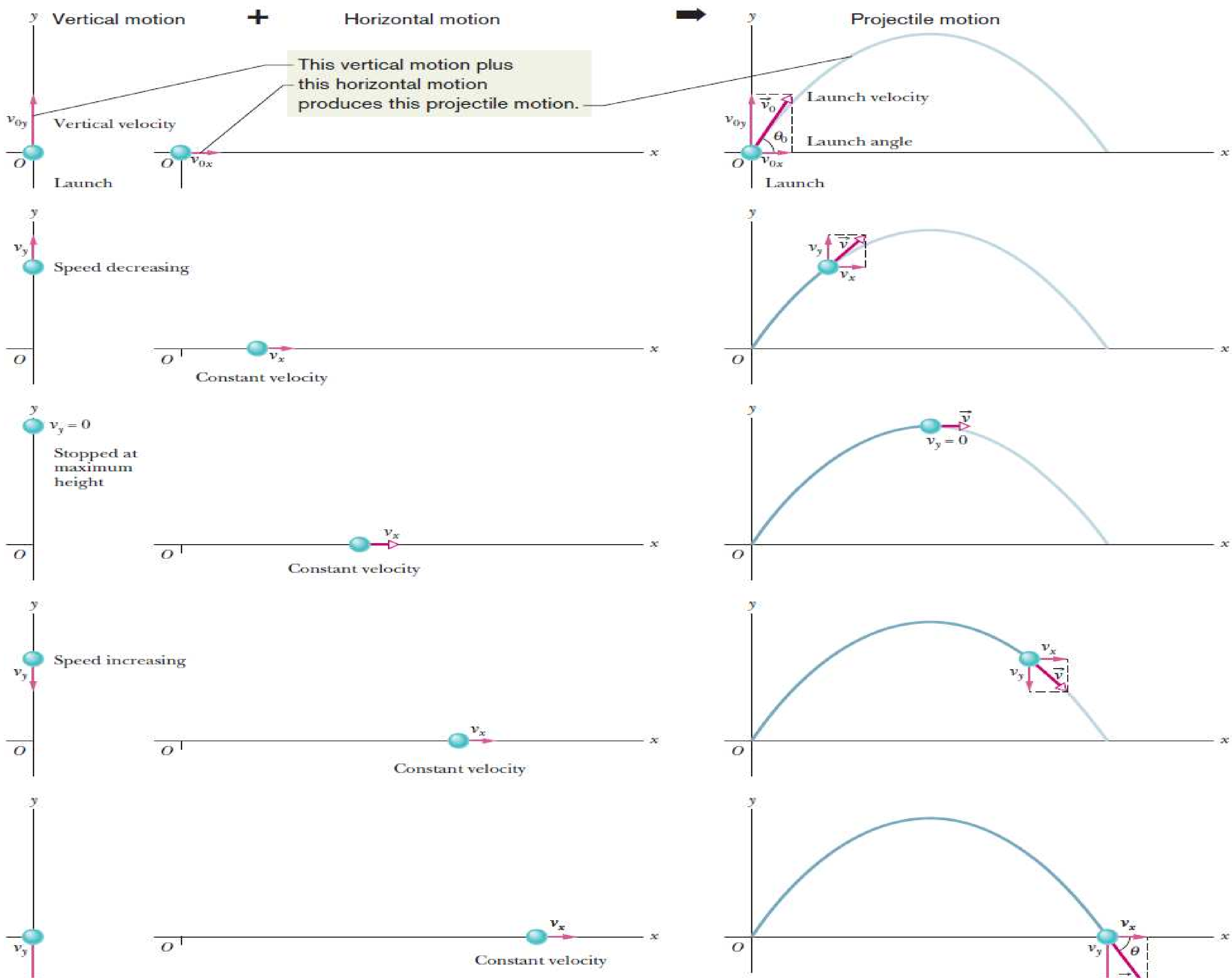
The initial velocity of the projectile is: $\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}.$

Here,

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0.$$

$$a_x = 0 \qquad a_y = -g$$




Vertical motion
Horizontal motion
Projectile motion
This vertical motion plus
this horizontal motion
produces this projectile motion.
Vertical velocity
Launch
Launch velocity
Launch angle
Launch
Speed decreasing
Constant velocity
Stopped at
maximum
height
Constant velocity
Speed increasing
Constant velocity
Constant velocity

Projectile Motion Analyzed

(assuming no external forces other than the weight)

Horizontal Motion: no acceleration

$$x - x_0 = v_{0x}t.$$

$$x - x_0 = (v_0 \cos \theta_0)t.$$

Vertical Motion; acceleration = -g

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$
$$= (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin \theta_0 - gt$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0).$$

By eliminating time, t:

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$

Horizontal Range (assuming no external forces)

The horizontal range of a projectile is the horizontal distance when it returns to its launching height.

The distance equations in the x- and y- directions respectively:

$$R = (v_0 \cos \theta_0)t$$

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2.$$

Eliminating t;

$$\Rightarrow \quad R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0. = \frac{v_0^2}{g} \sin 2\theta_0.$$

The horizontal range R is maximum for a launch angle of 45°.

The Effects of the Air

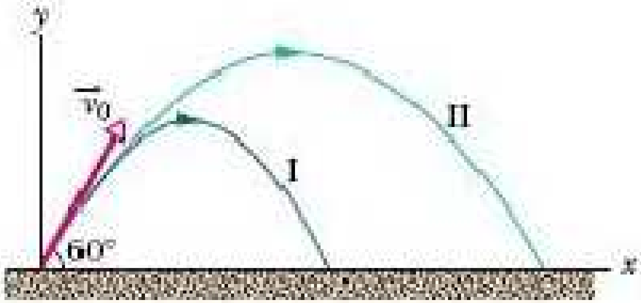


FIG. 4-14 (I) The path of a fly ball calculated by taking air resistance into account. (II) The path the ball would follow in a vacuum, calculated by the methods of this chapter. See Table 4-1 for corresponding data. (Adapted from "The Trajectory of a Fly Ball," by Peter J. Brancazio, *The Physics Teacher*, January 1985.)

TABLE 4-1

Two Fly Balls[a]

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.5 s	7.9 s

[a]See Fig. 4-14. The launch angle is 60° and the launch speed is 44.7 m/s.

CHECKPOINT

- A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity?
- What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent and at the topmost point of its flight?

Example 4: Projectile Motion (Projectile dropped from airplane)

In Fig. 4-14, a rescue plane flies at 198 km/h (= 55.0 m/s) and constant height $h = 500$ m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

KEY IDEAS

Once released, the capsule is a projectile, so its horizontal and vertical motions can be considered separately (we need not consider the actual curved path of the capsule).

Calculations: In Fig. 4-14, we see that ϕ is given by

$$\phi = \tan^{-1} \frac{x}{h}, \qquad (4\text{-}27)$$

where x is the horizontal coordinate of the victim (and of the capsule when it hits the water) and $h = 500$ m. We should be able to find x with Eq. 4-21:

$$x - x_0 = (v_0 \cos \theta_0)t. \qquad (4\text{-}28)$$

Here we know that $x_0 = 0$ because the origin is placed at the point of release. Because the capsule is *released* and not shot from the plane, its initial velocity $\vec{v}_0$ is equal to the plane's velocity. Thus, we know also that the initial velocity has magnitude $v_0 = 55.0$ m/s and angle $\theta_0 = 0°$ (measured relative to the positive direction of the x axis). However, we do not know the time t the capsule takes to move from the plane to the victim.

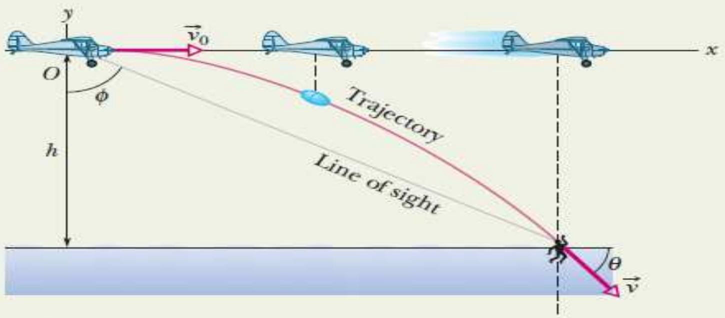


Fig. 4-14 A plane drops a rescue capsule while moving at constant velocity in level flight. While falling, the capsule remains under the plane.

To find t, we next consider the *vertical* motion and specifically Eq. 4-22:

$$y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2. \qquad (4\text{-}29)$$

Here the vertical displacement $y - y_0$ of the capsule is -500 m (the negative value indicates that the capsule moves *downward*). So,

$$-500 \text{ m} = (55.0 \text{ m/s})(\sin 0°)t - \tfrac{1}{2}(9.8 \text{ m/s}^2)t^2. \qquad (4\text{-}30)$$

Solving for t, we find $t = 10.1$ s. Using that value in Eq. 4-28 yields

$$x - 0 = (55.0 \text{ m/s})(\cos 0°)(10.1 \text{ s}), \qquad (4\text{-}31)$$

or

$$x = 555.5 \text{ m}.$$

Then Eq. 4-27 gives us

$$\phi = \tan^{-1} \frac{555.5 \text{ m}}{500 \text{ m}} = 48.0°. \qquad \text{(Answer)}$$

(b) As the capsule reaches the water, what is its velocity $\vec{v}$ in unit-vector notation and in magnitude-angle notation?

KEY IDEAS

(1) The horizontal and vertical components of the capsule's velocity are independent. (2) Component v_x does not change from its initial value $v_{0x} = v_0 \cos \theta_0$ because there is no horizontal acceleration. (3) Component v_y changes from its initial value $v_{0y} = v_0 \sin \theta_0$ because there is a vertical acceleration.

Calculations: When the capsule reaches the water,

$$v_x = v_0 \cos \theta_0 = (55.0 \text{ m/s})(\cos 0°) = 55.0 \text{ m/s}.$$

Using Eq. 4-23 and the capsule's time of fall $t = 10.1$ s, we also find that when the capsule reaches the water,

$$\begin{aligned} v_y &= v_0 \sin \theta_0 - gt \qquad (4\text{-}32)\\ &= (55.0 \text{ m/s})(\sin 0°) - (9.8 \text{ m/s}^2)(10.1 \text{ s}) \\ &= -99.0 \text{ m/s}. \end{aligned}$$

Thus, at the water

$$\vec{v} = (55.0 \text{ m/s})\hat{\text{i}} - (99.0 \text{ m/s})\hat{\text{j}}. \qquad \text{(Answer)}$$

Using Eq. 3-6 as a guide, we find that the magnitude and the angle of $\vec{v}$ are

$$v = 113 \text{ m/s} \quad \text{and} \quad \theta = -60.9°. \qquad \text{(Answer)}$$

Example 5: Projectile Motion (Cannonball to pirate ship)

Figure 4-15 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

KEY IDEAS

(1) A fired cannonball is a projectile. We want an equation that relates the launch angle θ_0 to the ball's horizontal displacement as it moves from cannon to ship. (2) Because the cannon and the ship are at the same height, the horizontal displacement is the range.

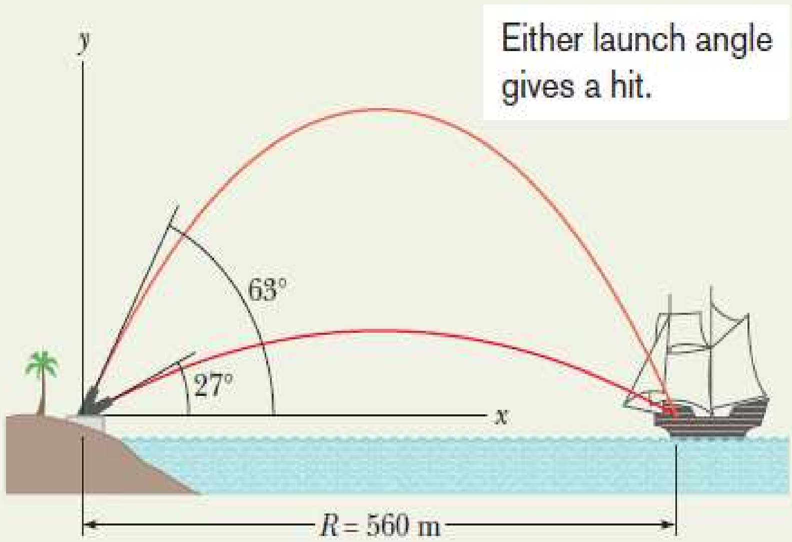


Fig. 4-15 A pirate ship under fire.

Calculations: We can relate the launch angle θ_0 to the range R with Eq. 4-26 which, after rearrangement, gives

$$\theta_0 = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} = \frac{1}{2}\sin^{-1}\frac{(9.8\text{ m/s}^2)(560\text{ m})}{(82\text{ m/s})^2}$$
$$= \frac{1}{2}\sin^{-1}0.816. \quad (4\text{-}33)$$

One solution of $\sin^{-1}$ (54.7°) is displayed by a calculator; we subtract it from 180° to get the other solution (125.3°). Thus, Eq. 4-33 gives us

$$\theta_0 = 27° \quad \text{and} \quad \theta_0 = 63°. \quad \text{(Answer)}$$

(b) What is the maximum range of the cannonballs?

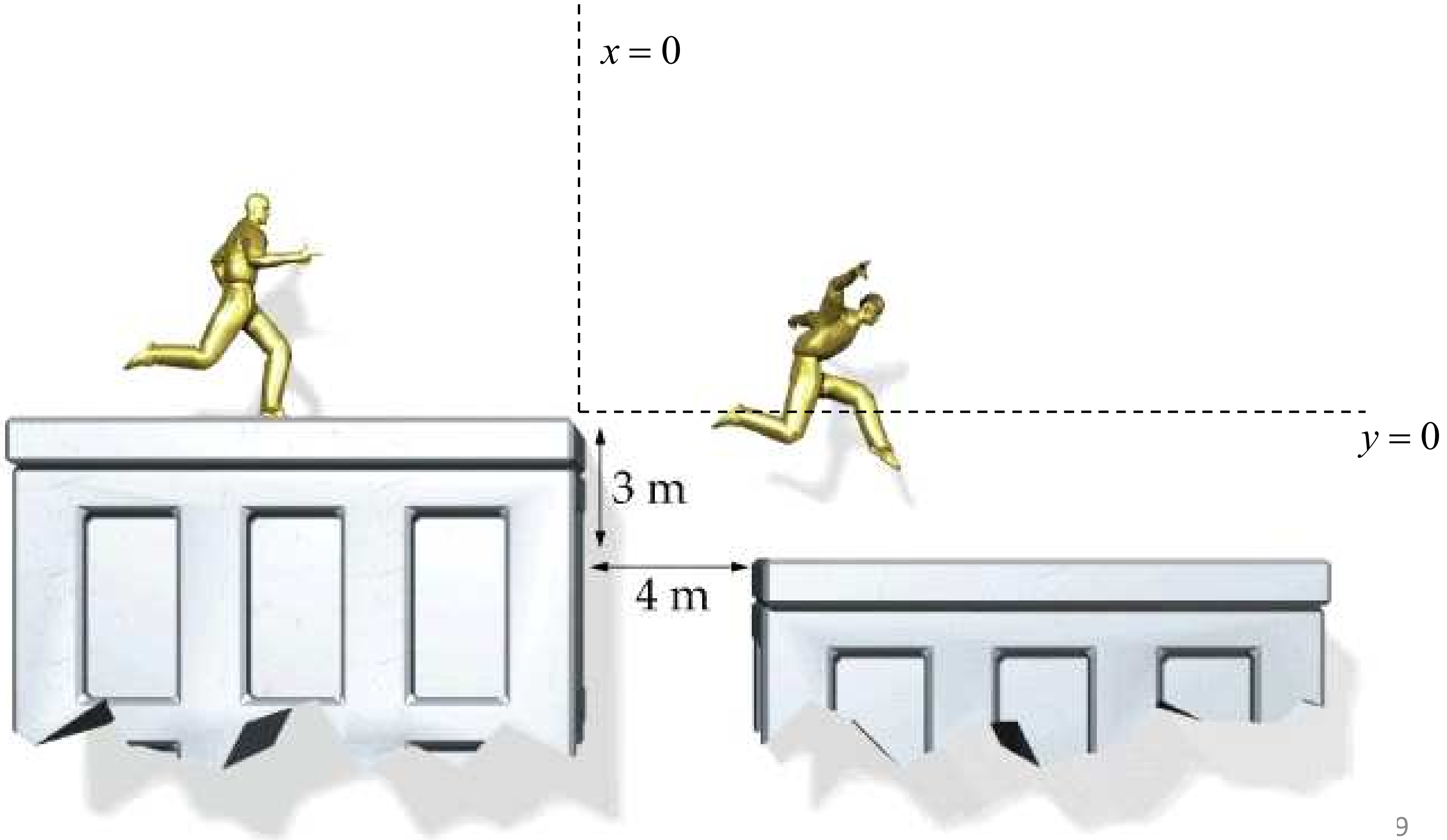
Calculations: We have seen that maximum range corresponds to an elevation angle θ_0 of 45°. Thus,

$$R = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(82\text{ m/s})^2}{9.8\text{ m/s}^2}\sin(2 \times 45°)$$
$$= 686\text{ m} \approx 690\text{ m}. \quad \text{(Answer)}$$

As the pirate ship sails away, the two elevation angles at which the ship can be hit draw together, eventually merging at $\theta_0 = 45°$ when the ship is 690 m away. Beyond that distance the ship is safe. However, the cannonballs could go farther if the cannon were higher.

Home Exercise

- A policeman chases a thief across city rooftops. They are both running at 5 m/s when they come to a gap between buildings that is 4 m wide and has a drop of 3 m.
- The thief leaps at 5 m/s at an angle of 45°. Does he clear the gap?
- The policeman leaps at 5 m/s horizontally. Does he clear the gap?



Uniform Circular Motion

The speed of the particle is constant

+

A particle travels around a circle/circular arc

→

Uniform circular motion

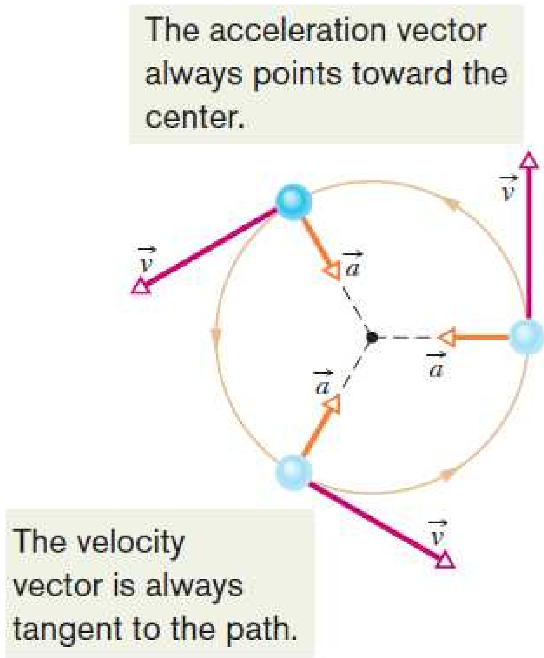
More on Uniform Circular Motion

As the direction of the velocity of the particle changes, there is an acceleration!!!

CENTRIPETAL (center-seeking) ACCELERATION

$$a = \frac{v^2}{r} \qquad \text{(centripetal acceleration),}$$

$$T = \frac{2\pi r}{v} \qquad \text{(period).}$$



! IMPORTANT NOTE !

When the motion is non-uniform circular motion, the speed and the direction change.

Centripetal acceleration, proof of a = v²/r

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v\sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}}.$$

$$\Rightarrow \quad \vec{v} = \left(-\frac{v y_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{v x_p}{r}\right)\hat{\mathrm{j}}.$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}.$$

Note

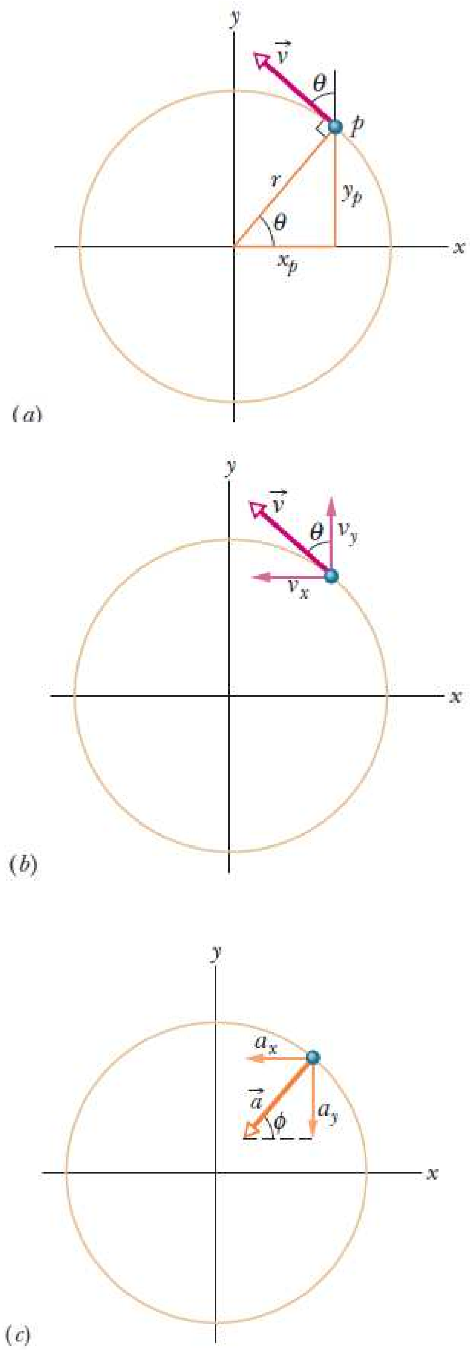
$$\frac{dy_P}{dt} = v_y, \quad \frac{dx_P}{dt} = v_x$$

$$v_x = -v\sin\theta, \quad v_y = -v\cos\theta$$

$$\Rightarrow \quad \vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}.$$

$$\Rightarrow \quad a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r},$$

$$\Rightarrow \quad \tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta.$$



Example 6: Uniform Circular motion (top gun pilots)

"Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g-LOC for "g-induced loss of consciousness."

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{\mathrm{i}} + 500\hat{\mathrm{j}})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{\mathrm{i}} - 500\hat{\mathrm{j}})$ m/s?

KEY IDEAS

We assume the turn is made with uniform circular motion.

Then the pilot's acceleration is centripetal and has magnitude *a given by* $a = v^2/R$.

Also, the time required to complete a full circle is the period given by $T = 2\pi R/v$

Calculations:

Because we do not know radius *R, let's solve for R* from the period equation for *R and substitute into the acceleration eqn.*

$$a = \frac{2\pi v}{T}.$$

Speed *v here is the (constant) magnitude of the velocity* during the turning.

$$v = \sqrt{(400\ \text{m/s})^2 + (500\ \text{m/s})^2} = 640.31\ \text{m/s}.$$

To find the period *T of the motion, first note that the final* velocity is the reverse of the initial velocity. This means the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given 24.0 s. Thus a full circle would have taken *T 48.0 s.*

Substituting these values into our equation for *a, we find*

$$a = \frac{2\pi(640.31\ \text{m/s})}{48.0\ \text{s}} = 83.81\ \text{m/s}^2 \approx 8.6g. \qquad \text{(Answer)}$$

Relative motion in one-dimension(1-D)

'The velocity of a particle depends on the reference frame of whoever is observing the velocity.'

- Suppose Alex (A) is at the origin of frame A (as in Fig.), watching car P (the "particle") speed past.
- Suppose Barbara (B) is at the origin of frame B, and is driving along the highway at constant speed, also watching car P. Suppose that they both measure the position of the car at a given moment. Then:

$$x_{PA} = x_{PB} + x_{BA}.$$

where x_{PA} is the position of P as measured by A. Consequently,

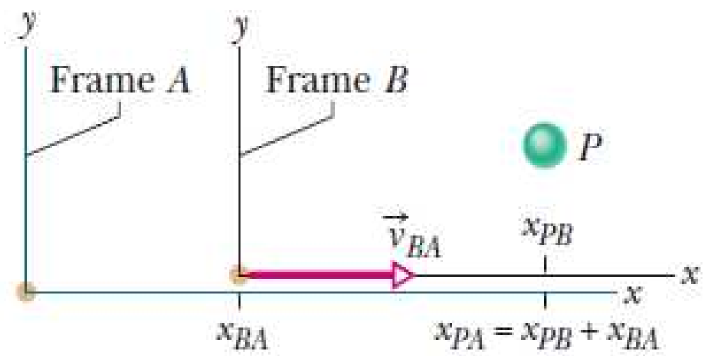
$$v_{PA} = v_{PB} + v_{BA}.$$

Also,

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA}).$$

Since v_{BA} *is constant, the last term is zero and we have*

$$\boldsymbol{a}_{PA} = \boldsymbol{a}_{PB}.$$



Example 7: Relative motion, 1-D (Barbara and Alex)

In Fig. 4-18, suppose that Barbara's velocity relative to Alex is a constant $v_{BA} = 52$ km/h and car P is moving in the negative direction of the x axis.

(a) If Alex measures a constant $v_{PA} = -78$ km/h for car P, what velocity v_{PB} will Barbara measure?

KEY IDEAS

We can attach a frame of reference A to Alex and a frame of reference B to Barbara. Because the frames move at constant velocity relative to each other along one axis, we can use Eq. 4-41 ($v_{PA} = v_{PB} + v_{BA}$) to relate v_{PB} to v_{PA} and v_{BA}.

Calculation: We find

$$-78 \text{ km/h} = v_{PB} + 52 \text{ km/h}.$$

Thus,

$$v_{PB} = -130 \text{ km/h}. \quad \text{(Answer)}$$

Comment: If car P were connected to Barbara's car by a cord wound on a spool, the cord would be unwinding at a speed of 130 km/h as the two cars separated.

(b) If car P brakes to a stop relative to Alex (and thus relative to the ground) in time $t = 10$ s at constant acceleration, what is its acceleration a_{PA} relative to Alex?

KEY IDEAS

To calculate the acceleration of car P *relative to Alex,* we must use the car's velocities *relative to Alex.* Because the acceleration is constant, we can use Eq. 2-11 ($v = v_0 + at$) to relate the acceleration to the initial and final velocities of P.

Calculation: The initial velocity of P relative to Alex is $v_{PA} = -78$ km/h and the final velocity is 0. Thus, the acceleration relative to Alex is

$$a_{PA} = \frac{v - v_0}{t} = \frac{0 - (-78 \text{ km/h})}{10 \text{ s}} \frac{1 \text{ m/s}}{3.6 \text{ km/h}}$$
$$= 2.2 \text{ m/s}^2. \quad \text{(Answer)}$$

(c) What is the acceleration a_{PB} of car P relative to Barbara during the braking?

KEY IDEA

To calculate the acceleration of car P *relative to Barbara,* we must use the car's velocities *relative to Barbara.*

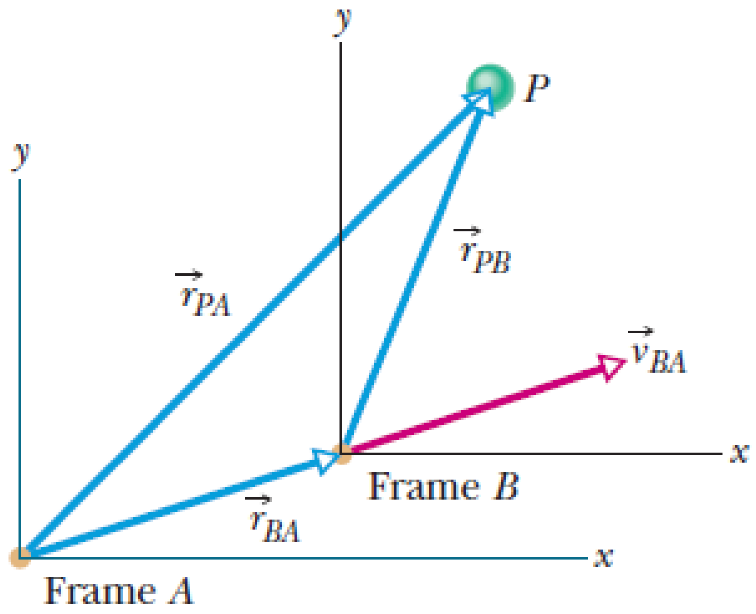
Calculation: We know the initial velocity of P relative to Barbara from part (a) ($v_{PB} = -130$ km/h). The final velocity of P relative to Barbara is -52 km/h (this is the velocity of the stopped car relative to the moving Barbara). Thus,

$$a_{PB} = \frac{v - v_0}{t} = \frac{-52 \text{ km/h} - (-130 \text{ km/h})}{10 \text{ s}} \frac{1 \text{ m/s}}{3.6 \text{ km/h}}$$
$$= 2.2 \text{ m/s}^2. \quad \text{(Answer)}$$

Comment: We should have foreseen this result: Because Alex and Barbara have a constant relative velocity, they must measure the same acceleration for the car.

Relative motion in two-dimensions(2-D)

A and B, the two observers, are watching P, the moving particle, from their origins of reference. B moves at a constant velocity with respect to A, while the corresponding axes of the two frames remain parallel. $\mathbf{r}_{PA}$ refers to the position of P as observed by A, and so on. From the situation, it is concluded:



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}.$$

↓

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}.$$

↓

$$\vec{a}_{PA} = \vec{a}_{PB}.$$

Example 8: Relative motion, 2-D (Airplanes)

In Fig. 4-20*a*, a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity $\vec{v}_{PW}$ relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?

KEY IDEAS

The situation is like the one in Fig. 4-19. Here the moving particle P is the plane, frame A is attached to the ground (call it G), and frame B is "attached" to the wind (call it W). We need a vector diagram like Fig. 4-19 but with three velocity vectors.

Calculations: First we construct a sentence that relates the three vectors shown in Fig. 4-20*b*:

velocity of plane relative to ground (*PG*)	=	velocity of plane relative to wind (*PW*)	+	velocity of wind relative to ground. (*WG*)

This relation is written in vector notation as

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}. \qquad (4\text{-}46)$$

We need to resolve the vectors into components on the coordinate system of Fig. 4-20*b* and then solve Eq. 4-46 axis by axis. For the *y* components, we find

$$v_{PG,y} = v_{PW,y} + v_{WG,y}$$

or $\quad 0 = -(215 \text{ km/h}) \sin\theta + (65.0 \text{ km/h})(\cos 20.0°).$

Solving for θ gives us

$$\theta = \sin^{-1}\frac{(65.0 \text{ km/h})(\cos 20.0°)}{215 \text{ km/h}} = 16.5°. \qquad \text{(Answer)}$$

Similarly, for the *x* components we find

$$v_{PG,x} = v_{PW,x} + v_{WG,x}.$$

Here, because $\vec{v}_{PG}$ is parallel to the *x* axis, the component $v_{PG,x}$ is equal to the magnitude v_{PG}. Substituting this notation and the value $\theta = 16.5°$, we find

$$v_{PG} = (215 \text{ km/h})(\cos 16.5°) + (65.0 \text{ km/h})(\sin 20.0°)$$
$$= 228 \text{ km/h}. \qquad \text{(Answer)}$$

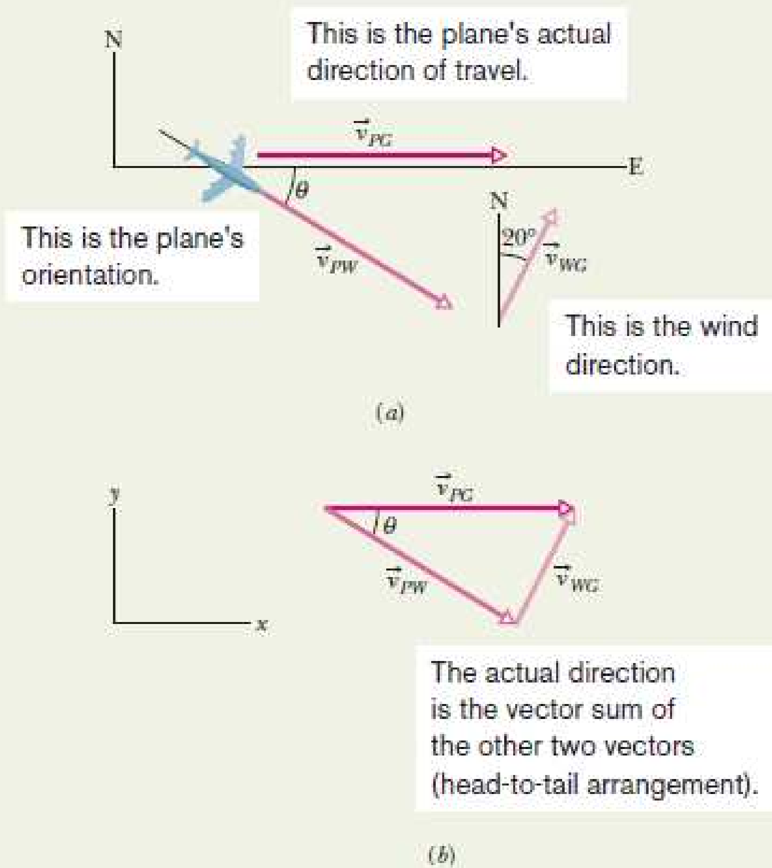


Fig. 4-20 A plane flying in a wind.