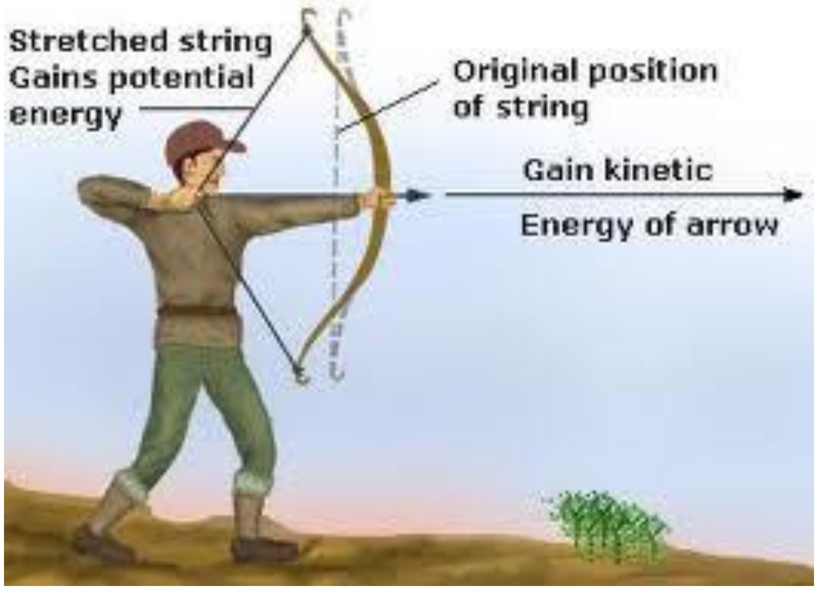
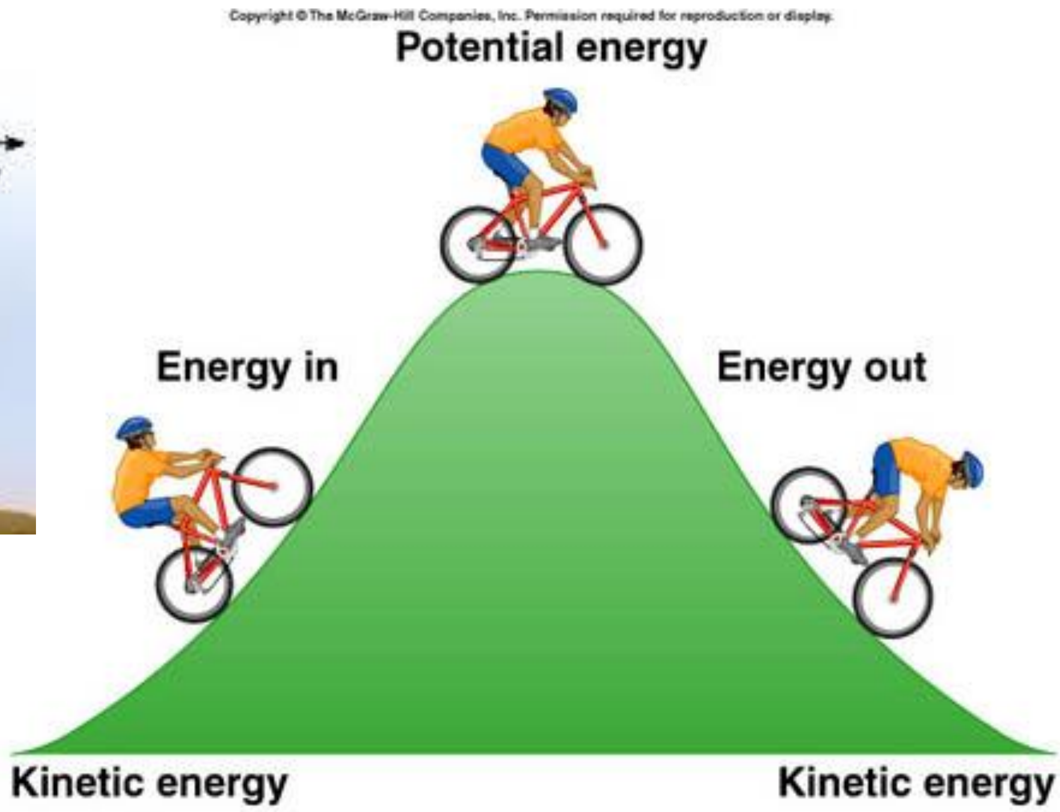
Chapter 8

Potential energy and conservation of energy



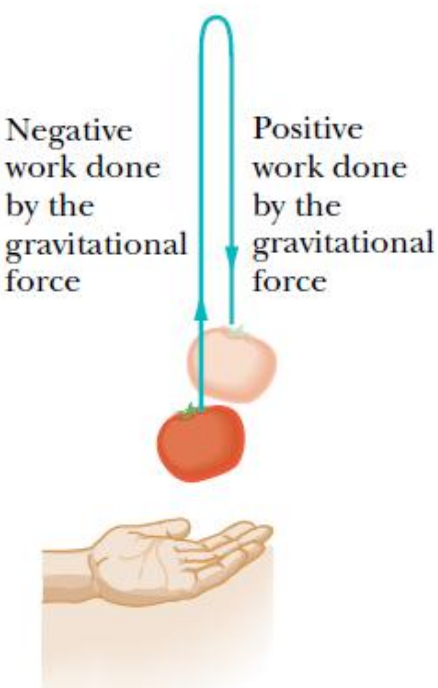


8.1 Potential energy

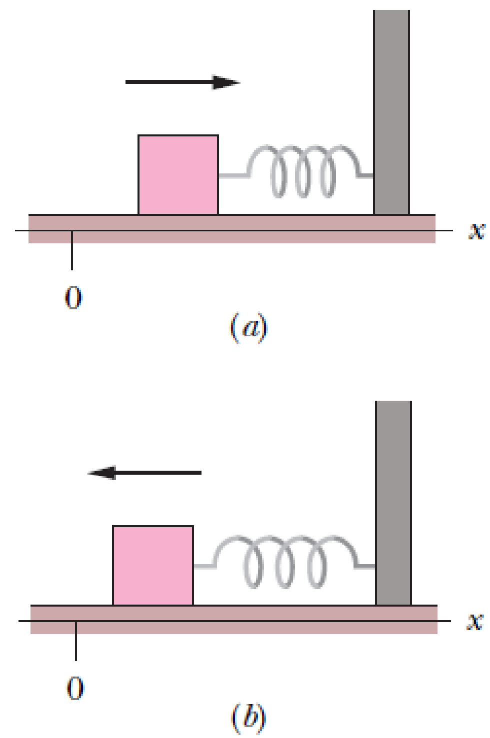
Energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

Some forms of potential energy:

Gravitational potential energy



Elastic potential energy



8.2 Work and potential energy

The change ΔU in potential energy (gravitational, elastic, etc) is defined as being equal to the negative of the work done on the object by the force (gravitational, elastic, etc)

$$\Delta U = -W.$$

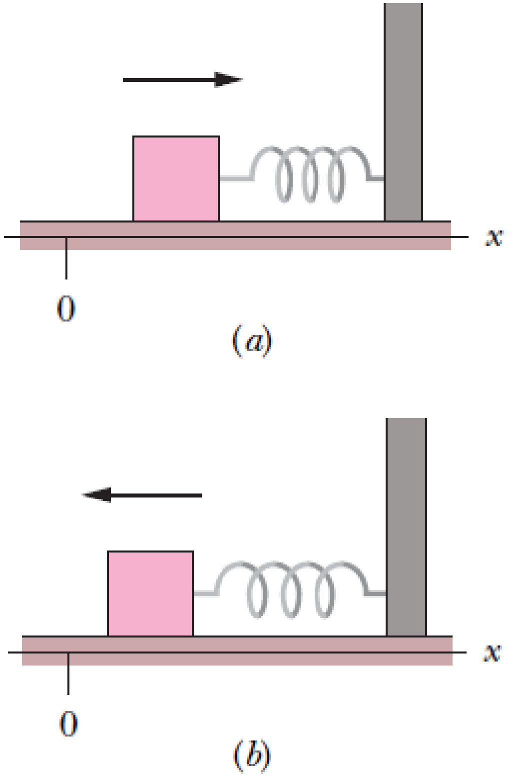


Fig. 8-3 A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (*a*) As the block moves rightward (as indicated by the arrow), the spring force does negative work on it. (*b*) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

8.2 Conservative and non-conservative forces

Suppose:

1. A system consists of two or more objects.

2. A force acts between a particle-like object in the system and the rest of the system.

3. When the system configuration changes, the force does work (call it W_1) on the object, transferring energy between the kinetic energy of the object, K, and some other type of energy of the system.

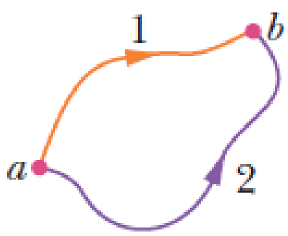
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation in which $W_1 = -W_2$ *is always true, the other type of energy is* a potential energy and the force is said to be a *<u>conservative force</u>*.

A force that is not conservative is called a *<u>non-conservative force</u>*. The kinetic frictional force and drag force are non-conservative.

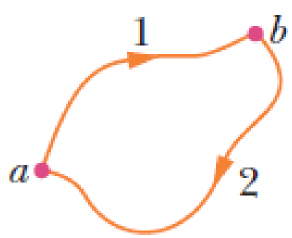
8.3 Path Independence of Conservative Forces

The net work done by a conservative force on a particle moving around any closed path is zero.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

$$W_{ab,1} = W_{ab,2},$$

If the work done from a to b along path 1 as $W_{ab,1}$ and the work done from b back to a along path 2 as $W_{ba,2}$. If the force is conservative, then the net work done during the round trip must be zero

$$W_{ab,1} + W_{ba,2} = 0,$$

$$W_{ab,1} = -W_{ba,2}.$$

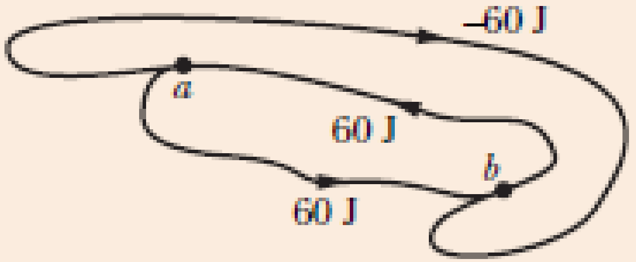
If the force is conservative,

$$W_{ab,2} = -W_{ba,2}.$$

$$\Rightarrow W_{ab,1} = W_{ab,2}$$

CHECKPOINT 1

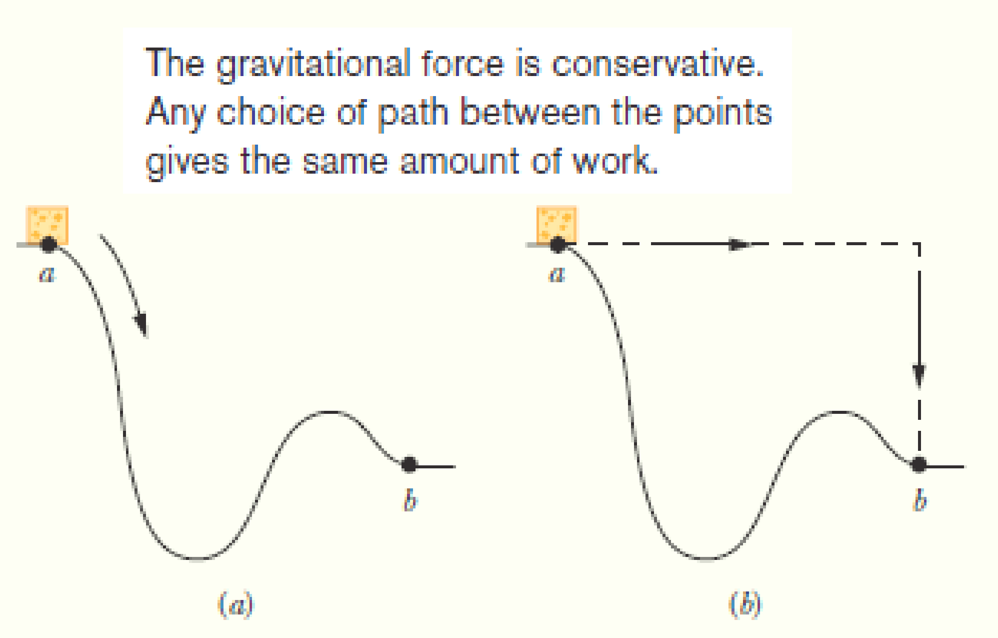
The figure shows three paths connecting points a and b. A single force $\vec{F}$ does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force $\vec{F}$ conservative?



Sample Problem

Equivalent paths for calculating work, slippery cheese

Figure 8-5*a* shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?



8.4: Determining Potential Energy values:

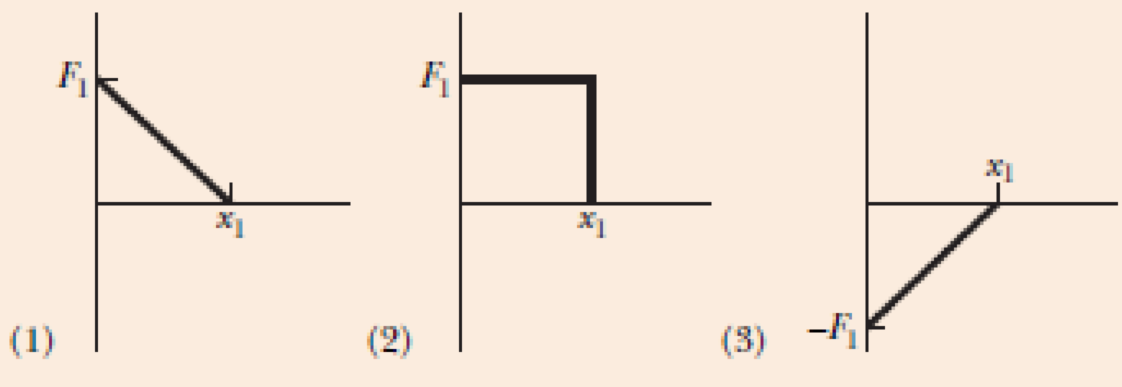
$$\Delta U = -W.$$

For the most general case, in which the force may vary with position, we may write the work *W* :

$$W = \int_{x_i}^{x_f} F(x)\,dx. \quad \Longrightarrow \quad \Delta U = -\int_{x_i}^{x_f} F(x)\,dx.$$

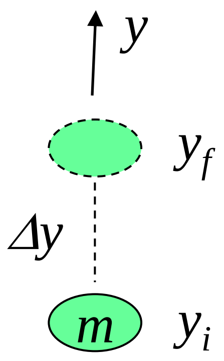
CHECKPOINT 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



8.4: Determining Potential Energy values:

Gravitational Potential Energy



A particle with mass *m* moving vertically along a *y* axis (the positive direction is upward).

The corresponding change in the gravitational potential energy of the particle–Earth system is:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\, dy = mg \int_{y_i}^{y_f} dy = mg\Big[\, y \,\Big]_{y_i}^{y_f},$$

$$\Longrightarrow \quad \Delta U = mg(y_f - y_i) = mg\,\Delta y.$$

$$U - U_i = mg(y - y_i).$$

$$\text{for } U_i = 0 \text{ and } y_i = 0 \quad \Longrightarrow \quad U(y) = \underbrace{mgy}_{\text{(gravitational potential energy)}}$$

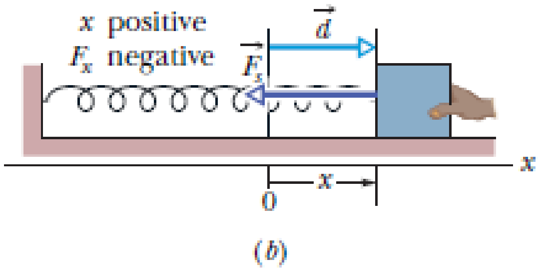
The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position y =0, not on the horizontal position.

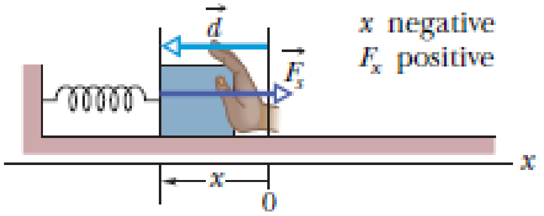
8.4: Determining Potential Energy values:

Elastic Potential Energy

In a block–spring system, the block is moving on the end of a spring of spring constant *k*.
The corresponding change in the elastic potential energy of the block–spring system is

$$\vec{F}_s = -k\vec{d}$$





$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f},$$

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

If the reference configuration is when the spring is at its relaxed length, and the block is at $x_i = 0$.

$$U - 0 = \tfrac{1}{2}kx^2 - 0, \Longrightarrow U(x) = \tfrac{1}{2}kx^2$$

(elastic potential energy).

Sample Problem

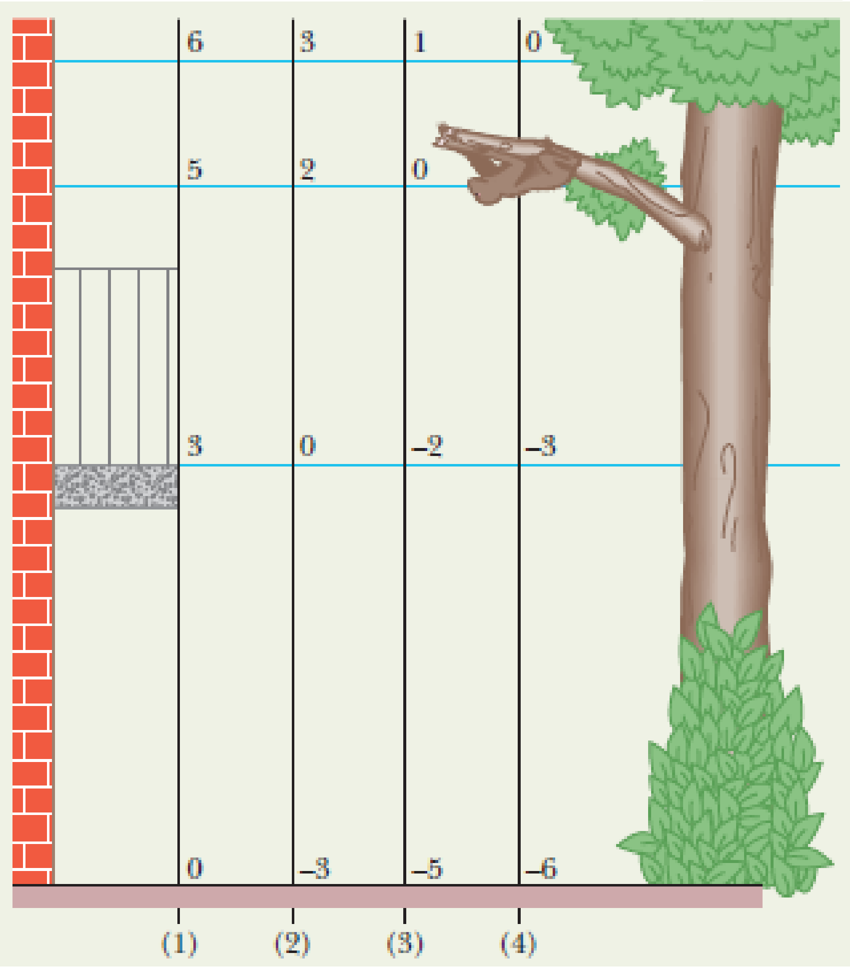
Choosing reference level for gravitational potential energy, sloth

A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 8-6).

(a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

Fig. 8-6



8.5: Conservation of Mechanical Energy

Principle of conservation of energy:

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

$$E_{\text{mec}} = K + U \qquad \text{(mechanical energy).}$$

system is *isolated* from its environment
no *external force*

$$\Delta K = W$$

$$\Delta U = -W.$$

$$\Delta K = -\Delta U.$$

$$K_2 - K_1 = -(U_2 - U_1),$$

$$K_2 + U_2 = K_1 + U_1 \qquad \text{(conservation of mechanical energy).}$$

$$\begin{pmatrix} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system} \end{pmatrix} = \begin{pmatrix} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system} \end{pmatrix},$$

Principle of conservation of mechanical energy

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.$$

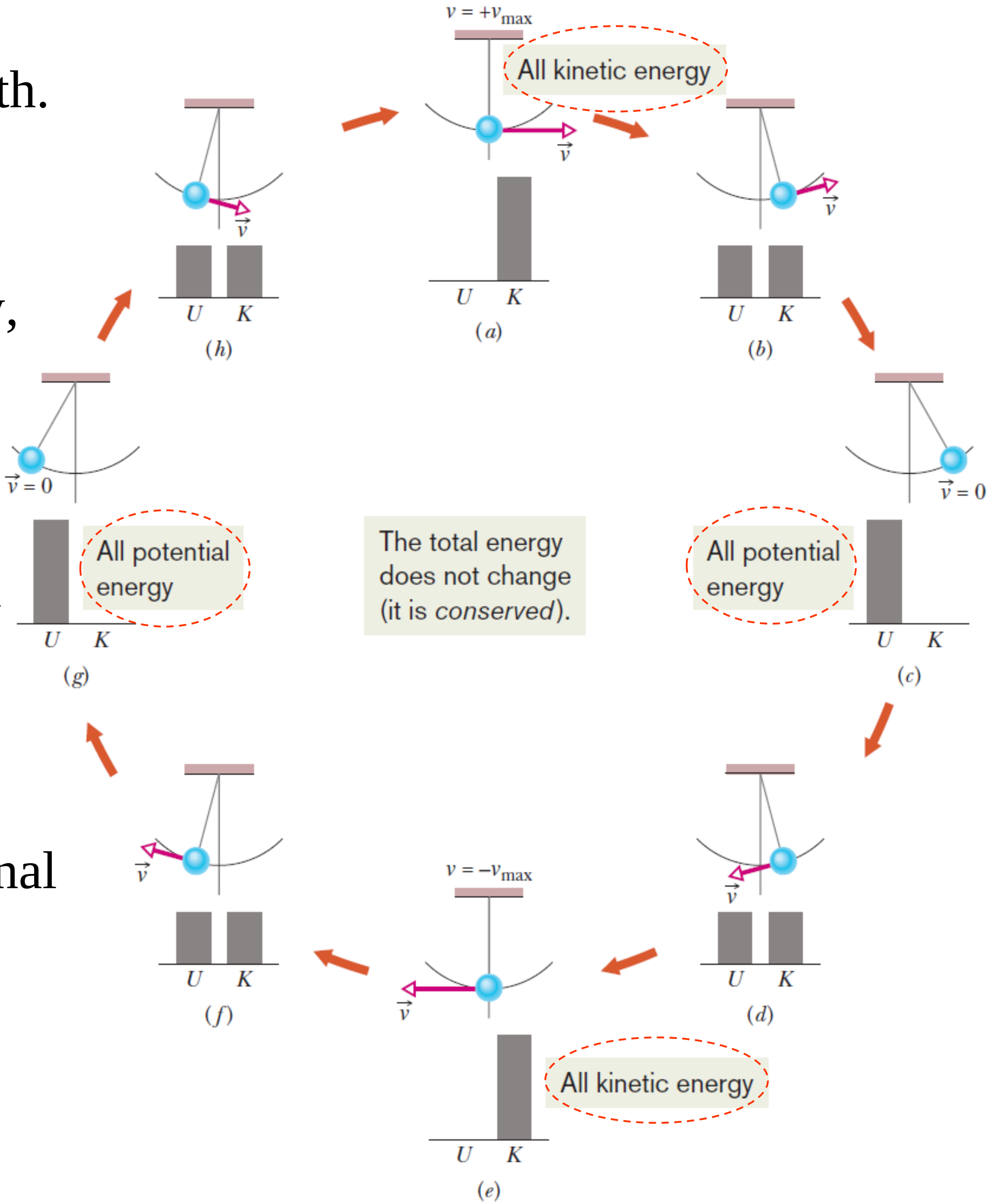
8.5: Conservation of Mechanical Energy

A pendulum swings back and forth.

Potential and kinetic energies of the pendulum– Earth system vary, but the mechanical energy E_{mec} of the system remains constant.

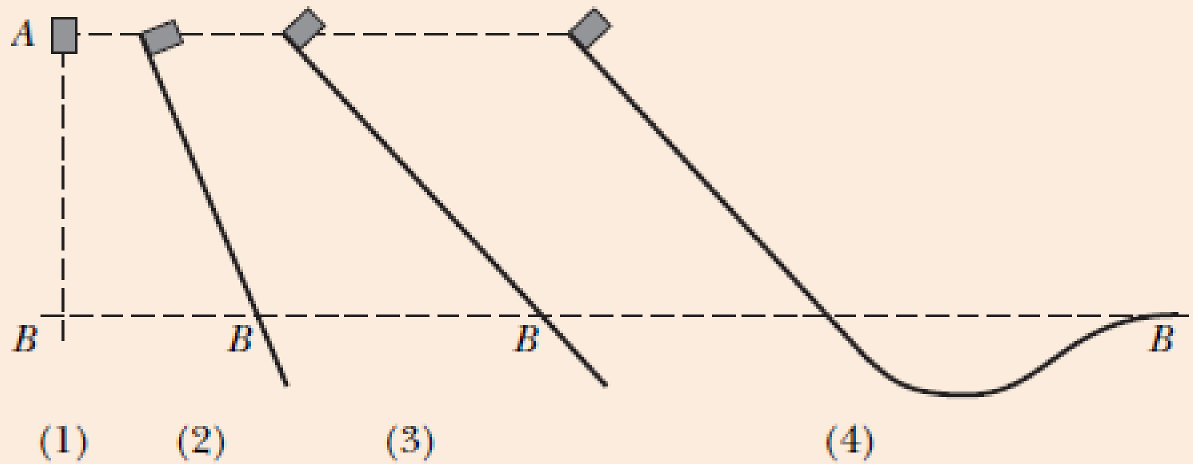
The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms.

If the swinging involved a frictional force then E_{mec} would not be conserved, and eventually the pendulum would stop.



CHECKPOINT 3

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point B, greatest first. (b) Rank them according to the speed of the block at point B, greatest first.



Sample Problem

Conservation of mechanical energy, water slide

In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

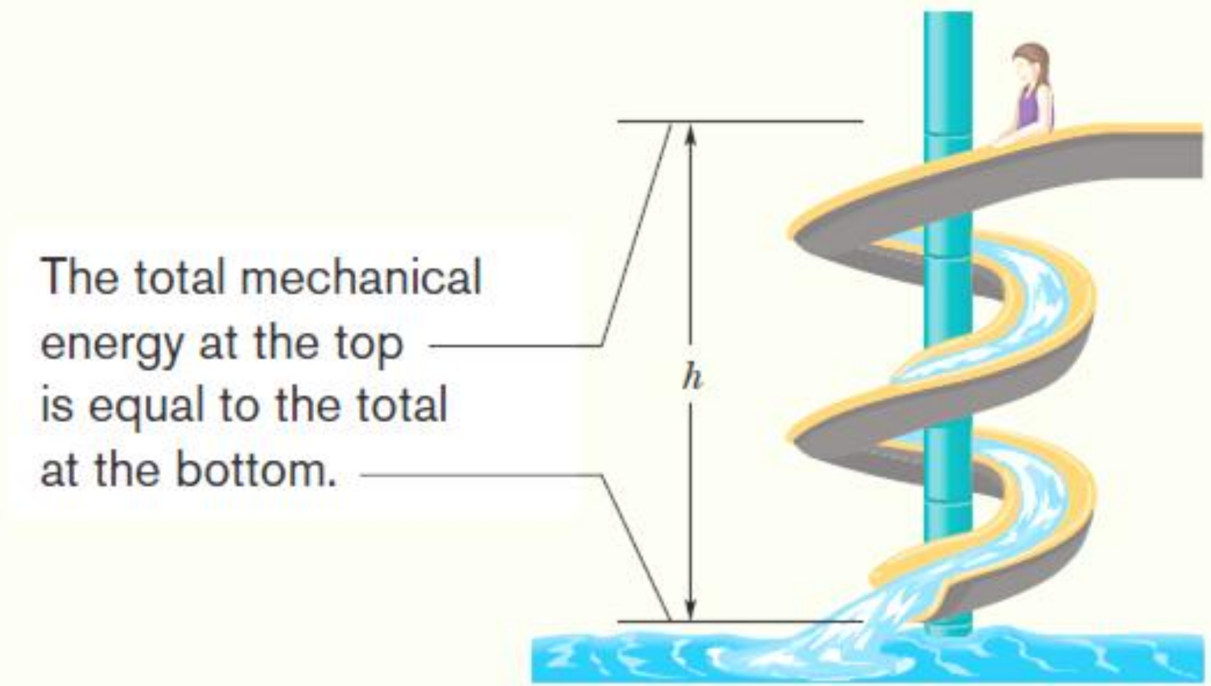


Fig. 8-8 A child slides down a water slide as she descends a height h.

8.6: Reading a Potential Energy Curve

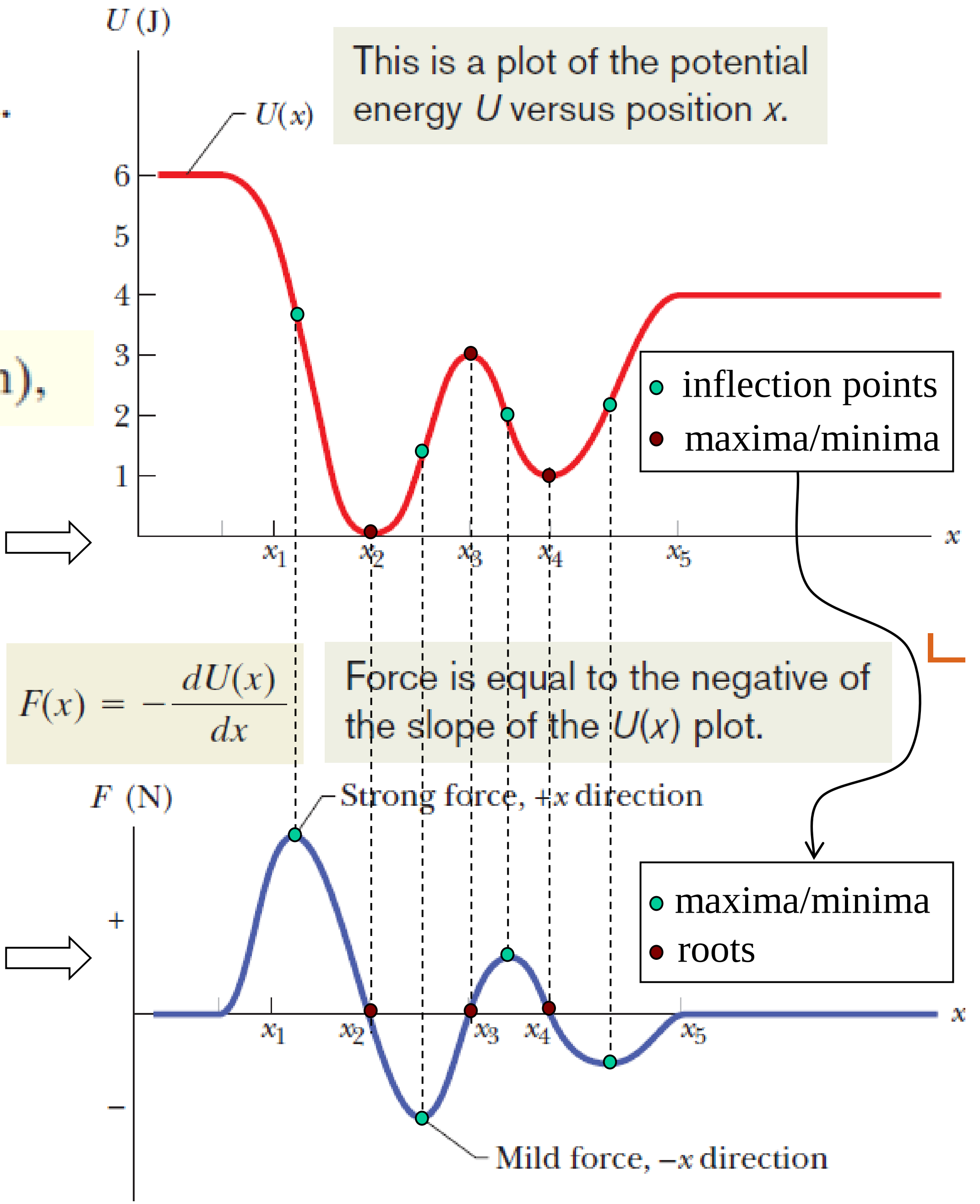
$$\Delta U(x) = -W = -F(x)\,\Delta x.$$

$$F(x) = -\frac{dU(x)}{dx}$$

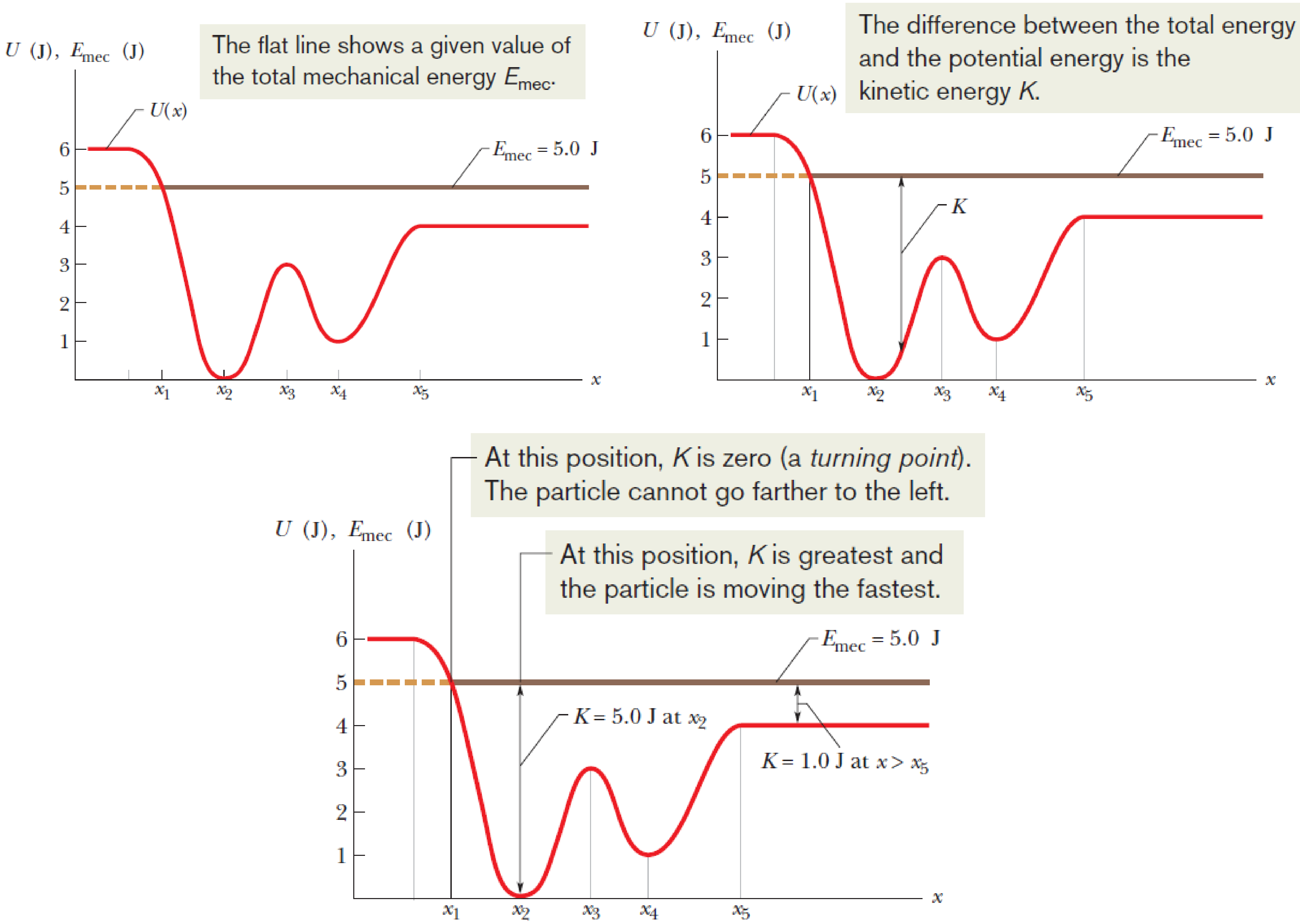
(one-dimensional motion),

A plot of *U(x)*, the potential energy function of a system containing a particle confined to move along an *x* axis. There is no friction, so mechanical energy is conserved.

A plot of the force *F(x)* acting on the particle, derived from the potential energy plot by taking its slope at various points.

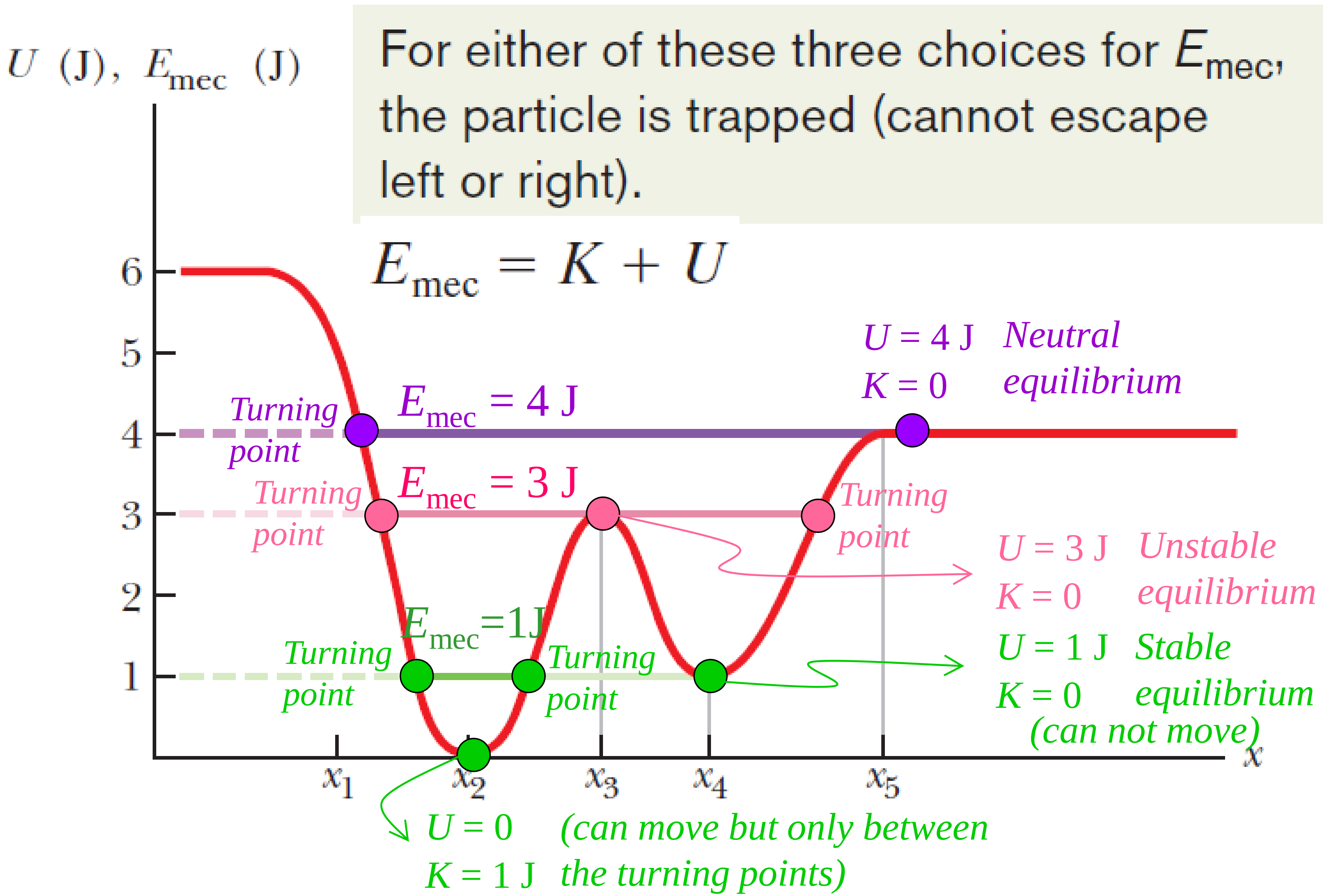


8.6: Reading a Potential Energy Curve



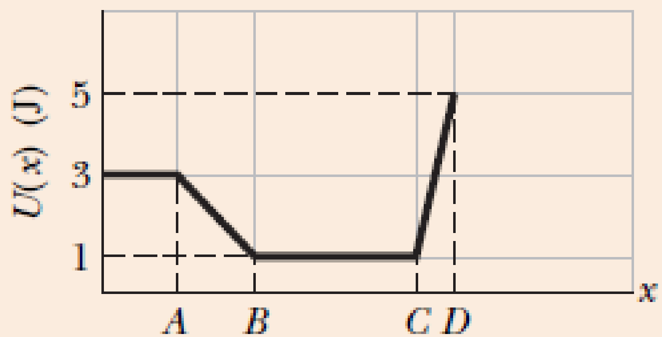
8.6: Potential Energy Curve, Equilibrium Points

The U(x) plot with three possible values of E_{mec}



CHECKPOINT 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. (a) Rank regions AB, BC, and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB?



Sample Problem

Reading a potential energy graph

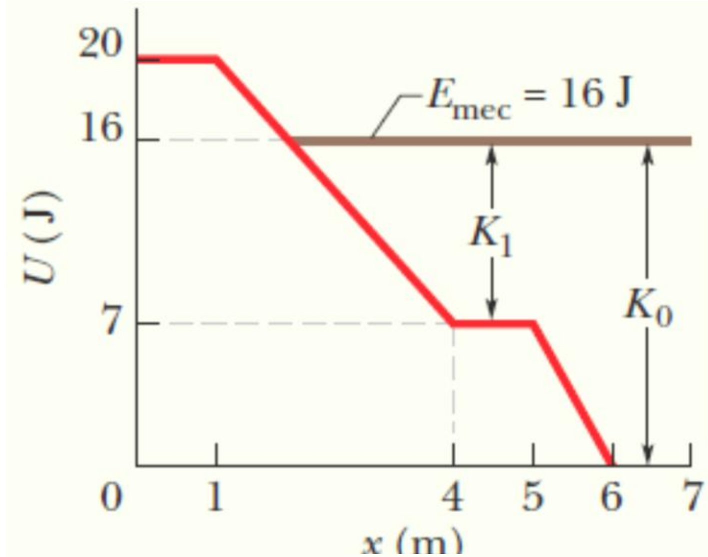
A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10*a*. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{\imath}$.

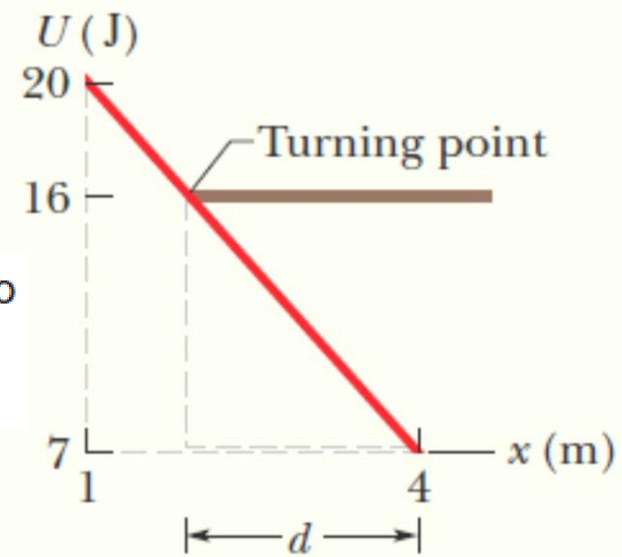
(a) From Fig. 8-10*a*, determine the particle's speed at $x_1 = 4.5$ m.

(b) Where is the particle's turning point located?

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0$ m.

Kinetic energy is the difference between the total energy and the potential energy.





The kinetic energy is zero at the turning point (the particle speed is zero).

8.7: Work done on a System by an External Force

Work is energy transferred to or from a system by means of an external force acting on that system.

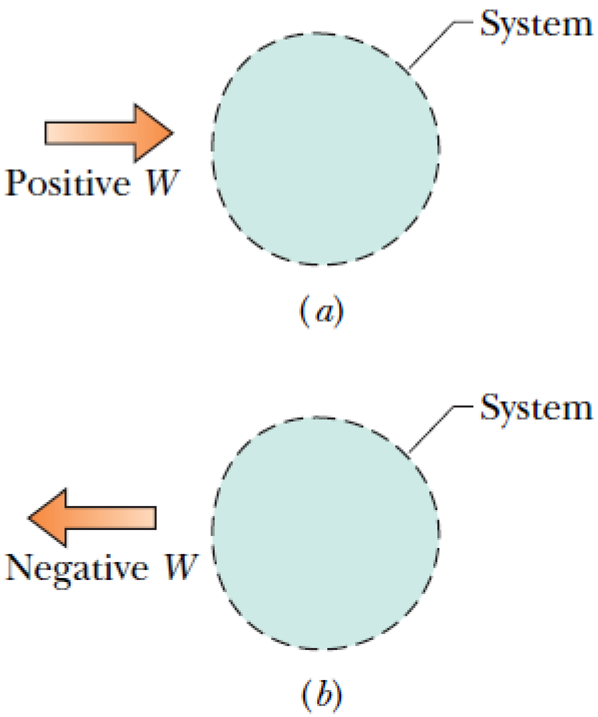


Fig. 8-11 (*a*) Positive work *W* done on an arbitrary system means a transfer of energy to the system. (*b*) Negative work *W* means a transfer of energy from the system.

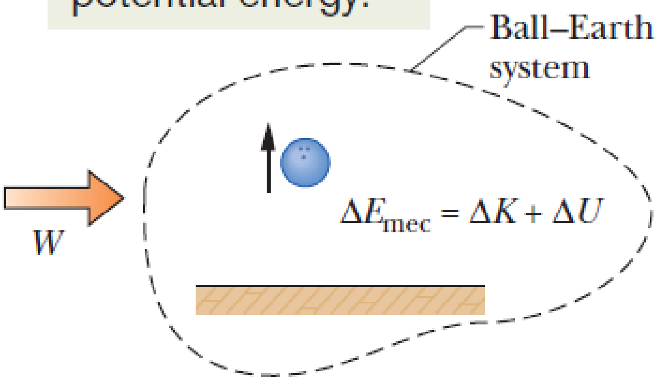
8.7: Work done on a System by an External Force

FRICTION NOT INVOLVED

$$W = \Delta K + \Delta U,$$

$$W = \Delta E_{mec}$$

Your lifting force transfers energy to kinetic energy and potential energy.



FRICTION INVOLVED

$$F - f_k = ma.$$

$$Fd = \Delta K + f_k d.$$

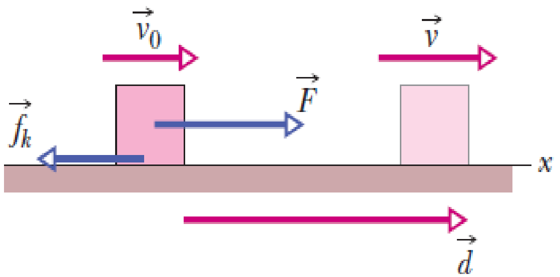
$$Fd = \Delta E_{mec} + f_k d.$$

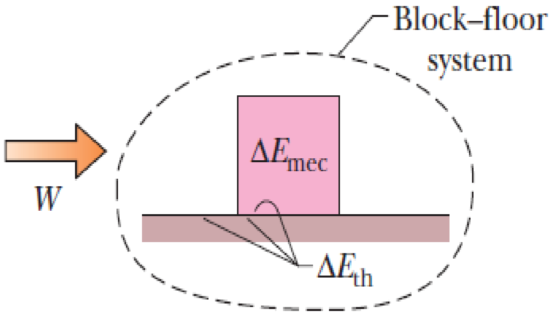
$$\Delta E_{th} = f_k d$$

$$W = \Delta E_{mec} + \Delta E_{th}$$

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

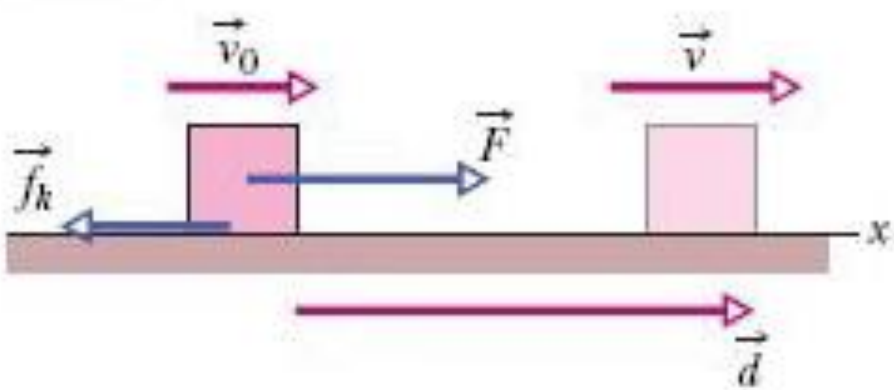




CHECKPOINT 5

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Fig. 8-13*a*. The magnitudes *F* of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance *d*. Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance *d*, greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases



Sample Problem

Work, friction, change in thermal energy, cabbage heads

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

KEY IDEA

$$W = \Delta E_{mec} + \Delta E_{th}.$$

Law of Conservation of Energy

The total energy *E* of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}}$$

where:
E_{mec} *is any change in the mechanical energy of the system,*
E_{th} *is any* change in the thermal energy of the system, and
E_{int} *is any change in any* other type of internal energy of the system.

The total energy *E* of an *isolated system* cannot change.

$$\Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}} = 0 \qquad \text{(isolated system)}$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

8.8: Conservation of Energy
External Forces and Internal Energy Transfers

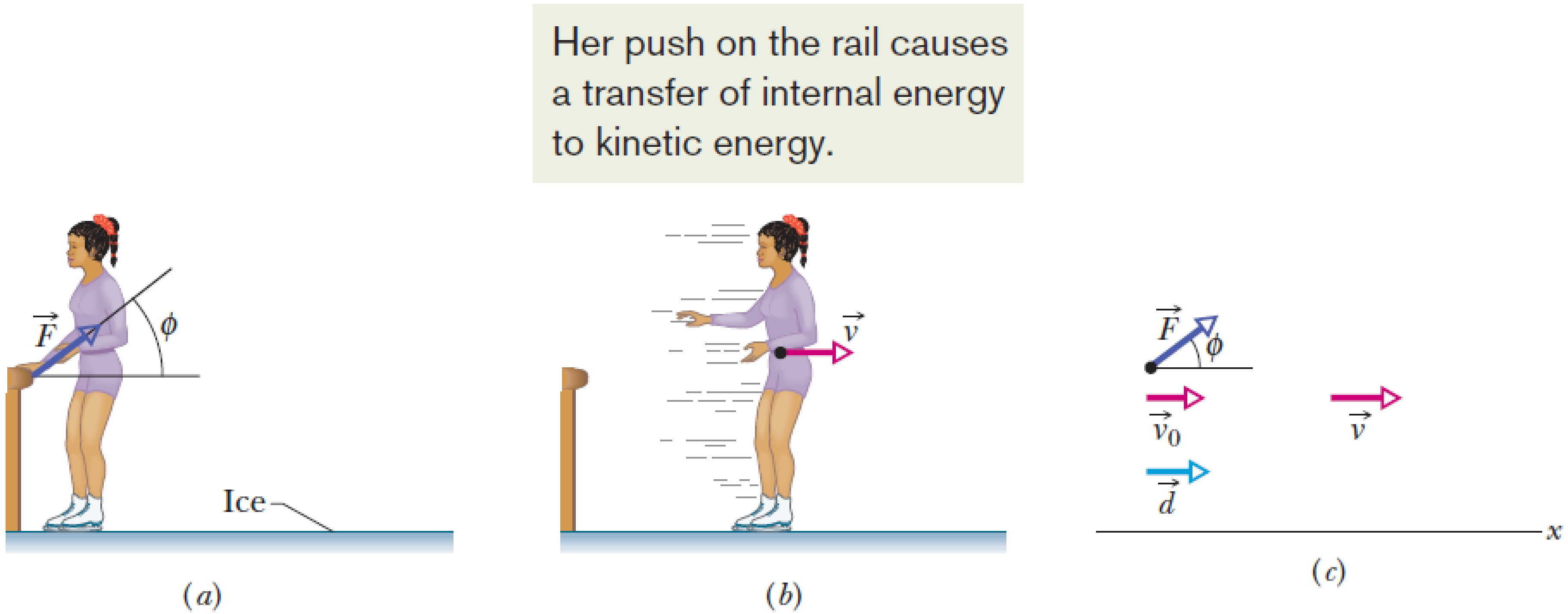


Fig. 8-15 (*a*) As a skater pushes herself away from a railing, the force on her from the railing is $\vec{F}$. (*b*) After the skater leaves the railing, she has velocity $\vec{v}$. (*c*) External force $\vec{F}$ acts on the skater, at angle ϕ with a horizontal x axis. When the skater goes through displacement $\vec{d}$, her velocity is changed from $\vec{v}_0$ (= 0) to $\vec{v}$ by the horizontal component of $\vec{F}$.

An external force can change the kinetic energy or potential energy of an object without doing work on the object, i.e. without transferring energy to the object. Instead, the force causes transfers of energy from one type to another

8.8: Conservation of Energy
External Forces and Internal Energy Transfers

$$K - K_0 = (F\cos\phi)d,$$

$$\Delta K = Fd\cos\phi.$$

$$\Delta U + \Delta K = Fd\cos\phi.$$

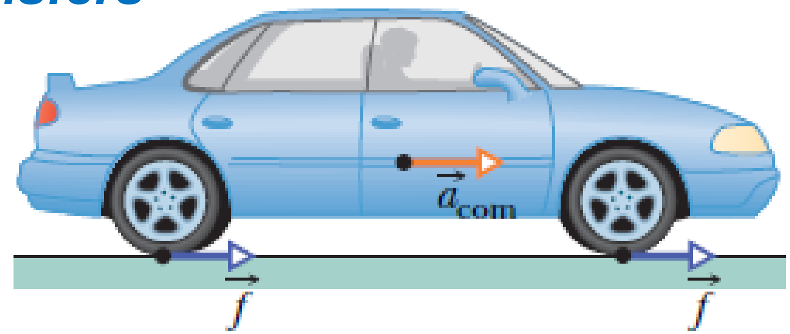


Fig. 8-16 A vehicle accelerates to the right using four-wheel drive. The road exerts four frictional forces (two of them shown) on the bottom surfaces of the tires. Taken together, these four forces make up the net external force $\vec{F}$ acting on the car.

8.8: Conservation of Energy: Power

In general, power *P* is the rate at which energy is transferred by a force from one type to another. If an amount of energy *E* is transferred in an amount of time t, the average power due to the force is

$$P_{avg} = \frac{\Delta E}{\Delta t}.$$

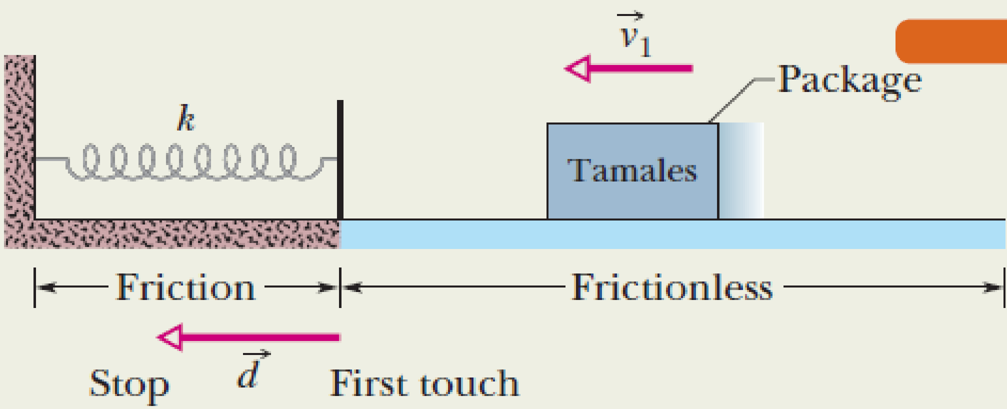
and the instantaneous power due to the force is

$$P = \frac{dE}{dt}.$$

Sample Problem

Energy, friction, spring, and tamales

In Fig. 8-17, a 2.0 kg package of tamales slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If $k = 10\,000$ N/m, by what distance d is the spring compressed when the package stops?



During the rubbing, kinetic energy is transferred to potential energy and thermal energy.

KEY IDEAS

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}}.$$