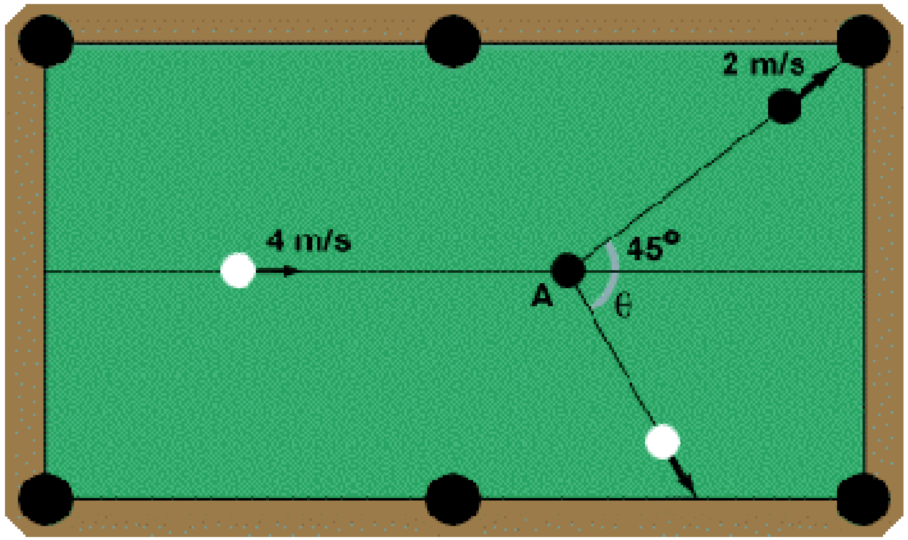
Chapter 9

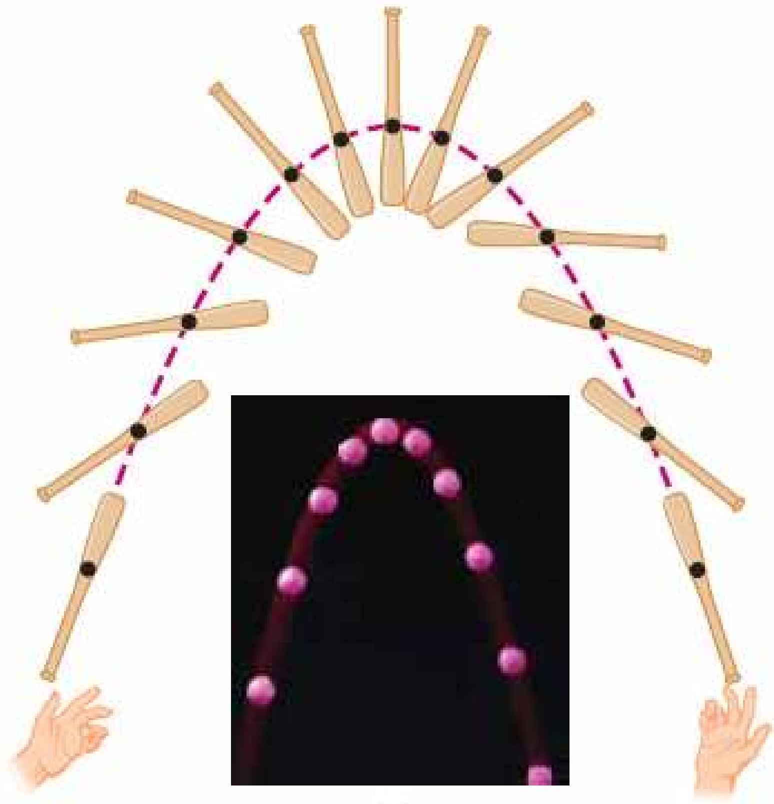
Center of Mass and Linear Momentum



The center of mass of a system of particles is the point that moves as though

(1) all of the system's mass were concentrated there and

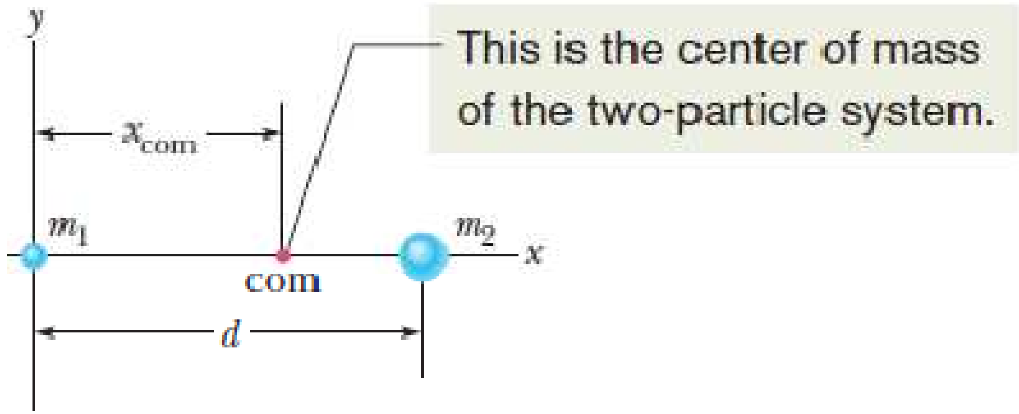
(2) all external forces were applied there.



The center of mass (black dot) of a baseball bat flipped into the air follows a parabolic path, but all other points of the bat follow more complicated curved paths.

9.2 The Center of Mass: A System of Particles

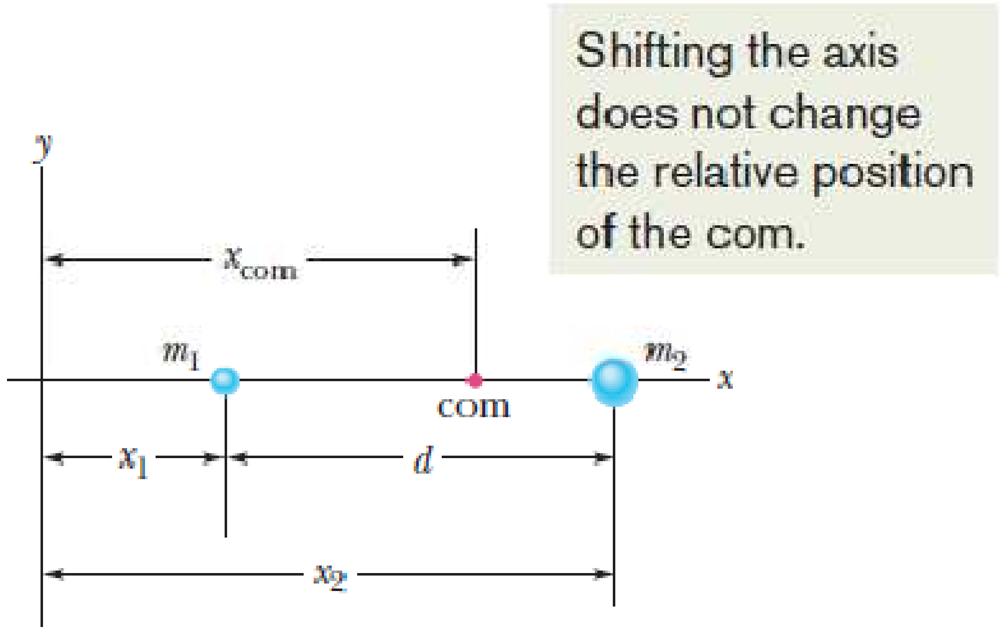
$$x_{com} = \frac{m_2}{m_1 + m_2} d.$$



$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{M},$$

$$M = m_1 + m_2.$$



9.2 The Center of Mass: A System of Particles

Consider a situation in which n particles are strung out along the x axis. Let the mass of the particles are m_1, m_2, ….m_n, and let them be located at x_1, x_2, …x_n respectively. Then if the total mass is $M = m_1 + m_2 + \ldots + m_n$, then the location of the center of mass, x_{com}, is

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M}$$

$$= \frac{1}{M} \sum_{i=1}^{n} m_i x_i .$$

9.2 The Center of Mass: A System of Particles

In 3-D, the locations of the center of mass are given by:

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

The position of the center of mass can be expressed in vector notation as:

$$\vec{r}_{\text{com}} = x_{\text{com}}\hat{\mathrm{i}} + y_{\text{com}}\hat{\mathrm{j}} + z_{\text{com}}\hat{\mathrm{k}}.$$

$$\Downarrow$$

$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i,$$

9.2 The Center of Mass: Solid Body

In the case of a solid body, the "particles" become differential mass elements dm, the sums become integrals, and the coordinates of the center of mass are defined as

$$x_{com} = \frac{1}{M}\int x\,dm, \qquad y_{com} = \frac{1}{M}\int y\,dm, \qquad z_{com} = \frac{1}{M}\int z\,dm$$

where M is the mass of the object.

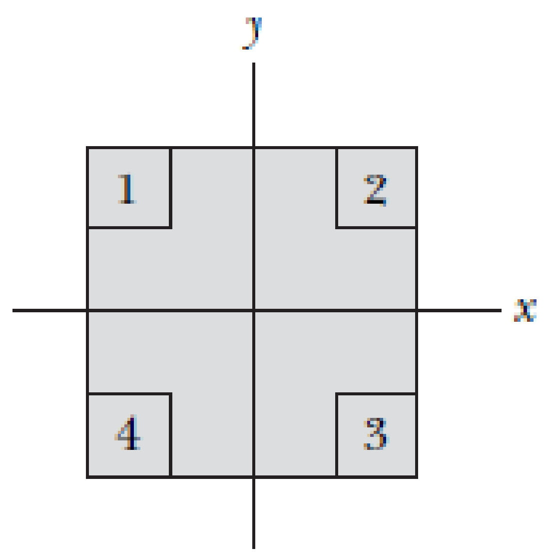
If the object has uniform density, r, defined as: $\rho = \frac{dm}{dV} = \frac{M}{V}.$

Then $$x_{com} = \frac{1}{V}\int x\,dV, \qquad y_{com} = \frac{1}{V}\int y\,dV, \qquad z_{com} = \frac{1}{V}\int z\,dV.$$

Where V is the volume of the object.

CHECKPOINT 1

The figure shows a uniform square plate from which four identical squares at the corners will be removed. (a) Where is the center of mass of the plate originally? Where is it after the removal of (b) square 1; (c) squares 1 and 2; (d) squares 1 and 3; (e) squares 1, 2, and 3; (f) all four squares? Answer in terms of quadrants, axes, or points (without calculation, of course).



Sample problem, COM

Figure 9-3*a* shows a uniform metal plate *P* of radius 2*R* from which a disk of radius *R* has been stamped out (removed) in an assembly line. The disk is shown in Fig. 9-3b. Using the *xy* coordinate system shown, locate the center of mass com_P of the remaining plate.

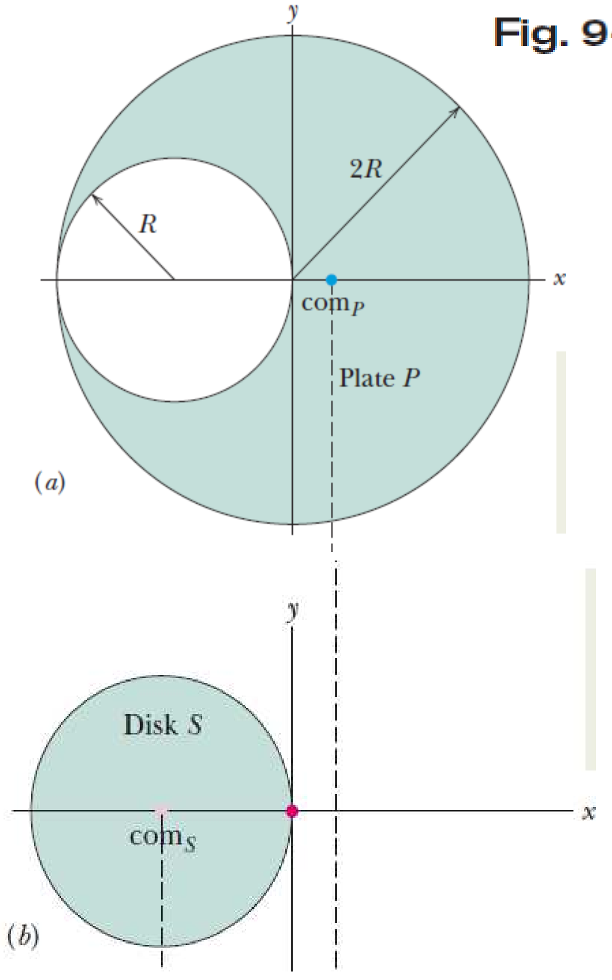
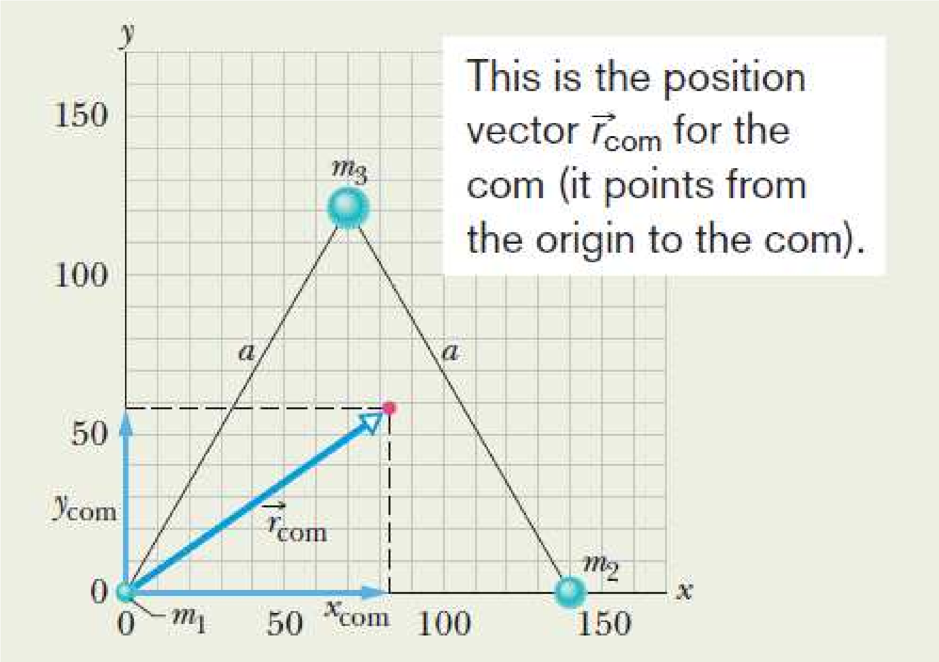


Fig. 9-3

Sample problem, COM of 3 particles

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?



We are given the following data:

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

9.3: Newton's 2nd Law for a System of Particles

The vector equation that governs the motion of the center of mass of such a system of particles is:

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} \quad \text{(system of particles).}$$

$$\Rightarrow \quad F_{\text{net},x} = Ma_{\text{com},x} \qquad F_{\text{net},y} = Ma_{\text{com},y} \qquad F_{\text{net},z} = Ma_{\text{com},z}.$$

Note that:

1. F_{net} is the net force of all external forces that act on the system. Forces on one part of the system from another part of the system (internal forces) are not included

2. M is the total mass of the system. M remains constant, and the system is said to be closed.

3. a_{com} is the acceleration of the center of mass of the system.

The internal forces of the explosion cannot change the path of the com.

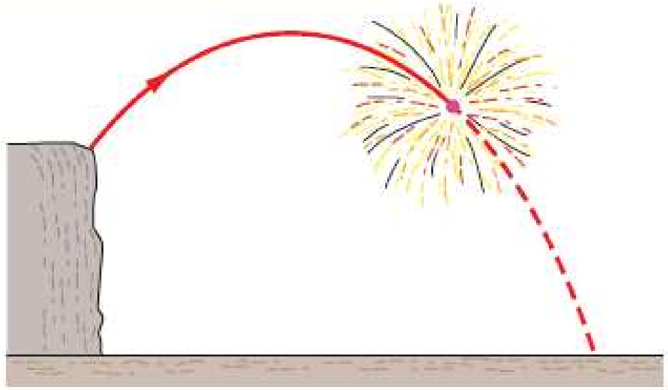


Fig. 9-5 A fireworks rocket explodes in flight. In the absence of air drag, the center of mass of the fragments would continue to follow the original parabolic path, until fragments began to hit the ground.

CHECKPOINT 2

Two skaters on frictionless ice hold opposite ends of a pole of negligible mass. An axis runs along it, with the origin at the center of mass of the two-skater system. One skater, Fred, weighs twice as much as the other skater, Ethel. Where do the skaters meet if (a) Fred pulls hand over hand along the pole so as to draw himself to Ethel, (b) Ethel pulls hand over hand to draw herself to Fred, and (c) both skaters pull hand over hand?

9.3: Newton's 2nd Law for a System of Particles: Proof of final result

- For a system of n particles, $$M\vec{r}_{com} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n,$$

where M is the total mass, and $\mathbf{r}_i$ are the position vectors of the masses m_i.

- Differentiating, $$M\vec{v}_{com} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.$$

where the **v** vectors are velocity vectors.

- This leads to $$M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n.$$

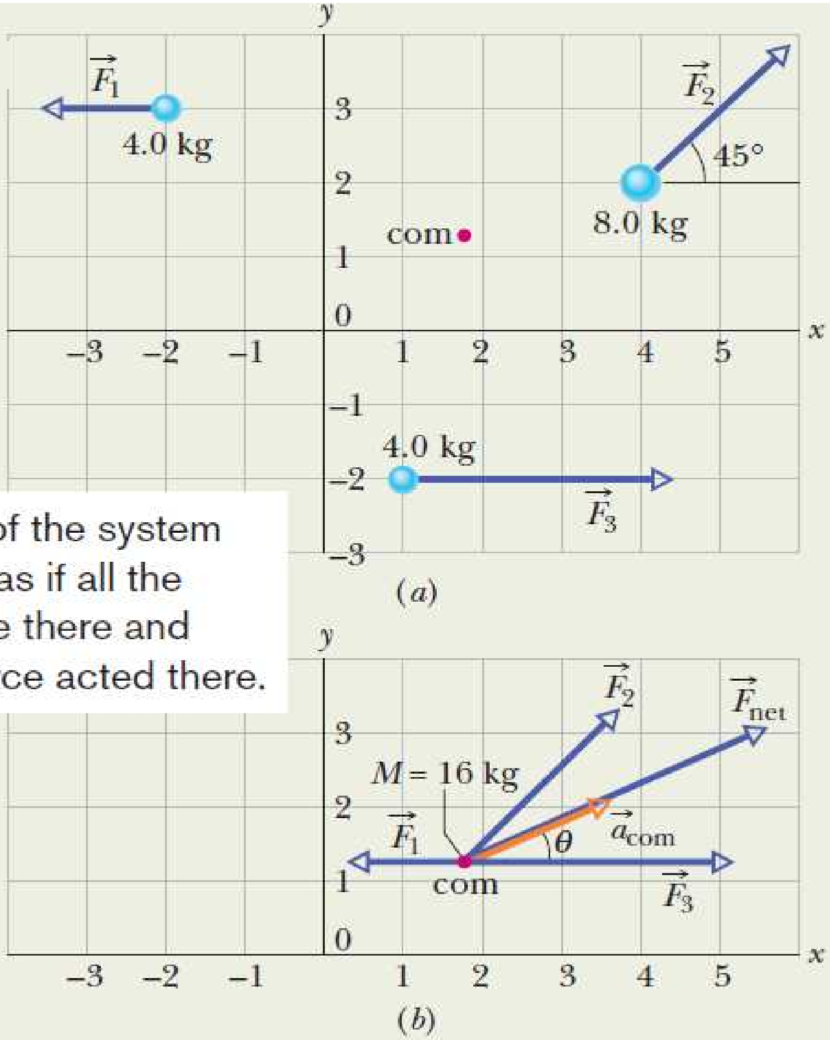
- Finally, $$M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n.$$

What remains on the right hand side is the vector sum of all the external forces that act on the system, while the internal forces cancel out by Newton's 3rd Law.

Sample problem: motion of the com of 3 particles

The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?

Fig. 9-7



The com of the system will move as if all the mass were there and the net force acted there.

9.4: Linear momentum

DEFINITION:

$$\vec{p} = m\vec{v} \qquad \text{(linear momentum of a particle)}$$

in which m is the mass of the particle and **v** is its velocity.

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

Manipulating this equation:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$ (Newton's 2^{nd} Law)

9.5: Linear Momentum of a System of Particles

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

$$\vec{P} = M\vec{v}_{\text{com}} \qquad \text{(linear momentum, system of particles),}$$

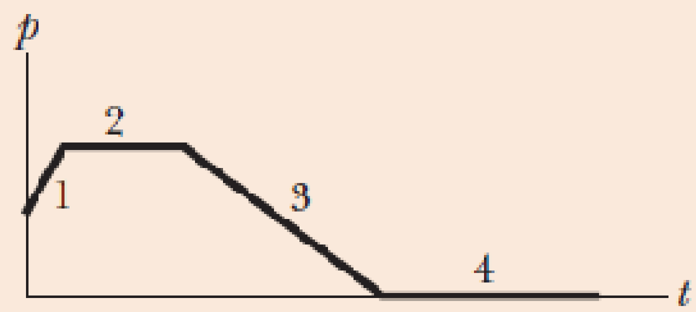
↓

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \qquad \text{(system of particles),}$$

CHECKPOINT 3

The figure gives the magnitude p of the linear momentum versus time t for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the magnitude of the force, greatest first. (b) In which region is the particle slowing?



9.6: Collision and Impulse

The collision of a ball with a bat collapses part of the ball. *(Photo by Harold E. Edgerton. ©The Harold and Esther Edgerton Family Trust, courtesy of Palm Press, Inc.)*

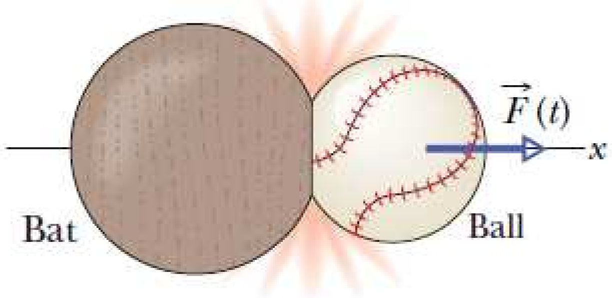


Fig. 9-8 Force $\vec{F}(t)$ acts on a ball as the ball and a bat collide.

In this case, the collision is brief, and the ball experiences a force that is great enough to slow, stop, or even reverse its motion.

The figure depicts the collision at one instant. The ball experiences a force F(t) that varies during the collision and changes the linear momentum of the ball.

9.6: Collision and Impulse

The change in linear momentum is related to the force by Newton's second law written in the form

$$\vec{F} = d\vec{p}/dt.$$

$$\longrightarrow \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\, dt.$$

$$\longrightarrow \vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt \qquad \text{(impulse defined).}$$

The right side of the equation is a measure of both the magnitude and the duration of the collision force, and is called the ***impulse of the collision*, J.**

9.6: Collision and Impulse

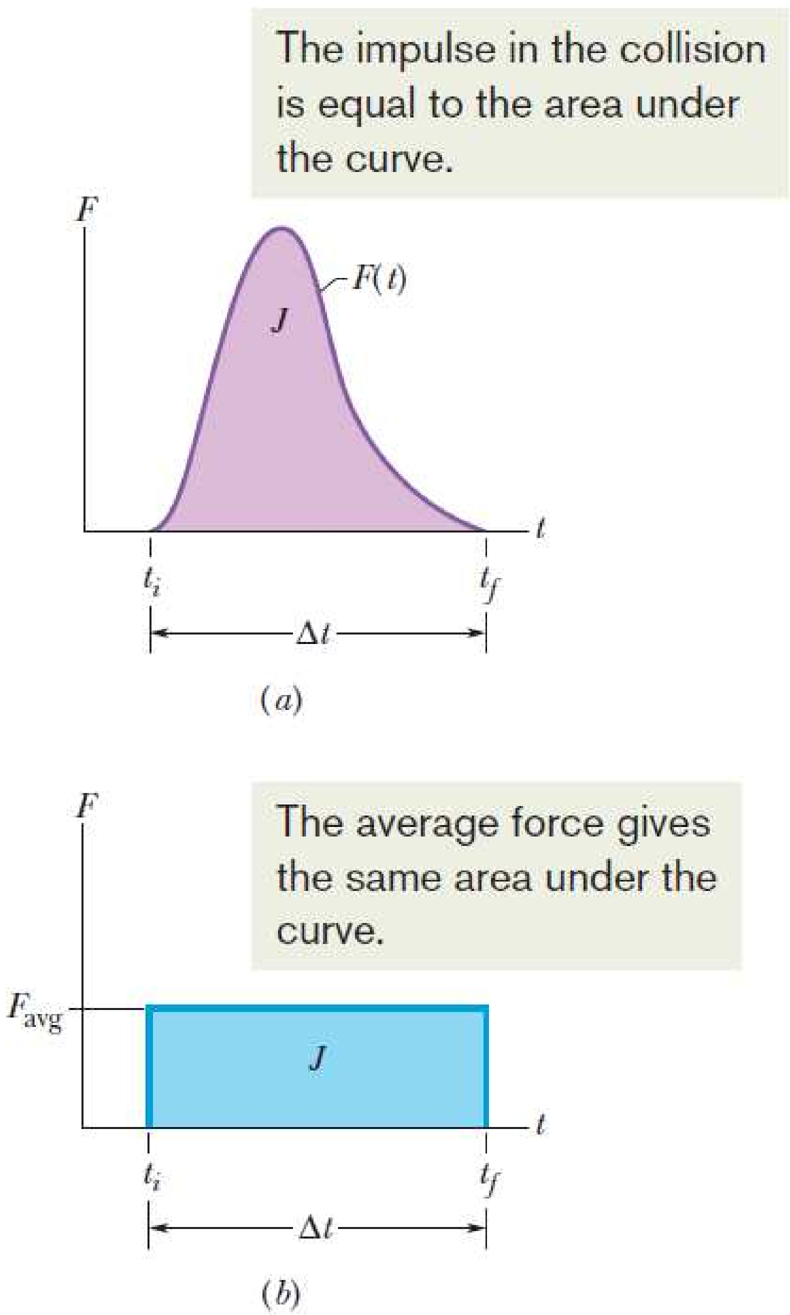


Fig. 9-9 (*a*) The curve shows the magnitude of the time-varying force $F(t)$ that acts on the ball in the collision of Fig. 9-8. The area under the curve is equal to the magnitude of the impulse $\vec{J}$ on the ball in the collision. (*b*) The height of the rectangle represents the average force F_{avg} acting on the ball over the time interval Δt. The area within the rectangle is equal to the area under the curve in (*a*) and thus is also equal to the magnitude of the impulse $\vec{J}$ in the collision.

Instead of the ball, one can focus on the bat. At any instant, Newton's third law says that the force on the bat has the same magnitude but the opposite direction as the force on the ball.

That means that the impulse on the bat has the same magnitude but the opposite direction as the impulse on the ball.

CHECKPOINT 4

A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

9.6: Collision and Impulse: Series of Collisions

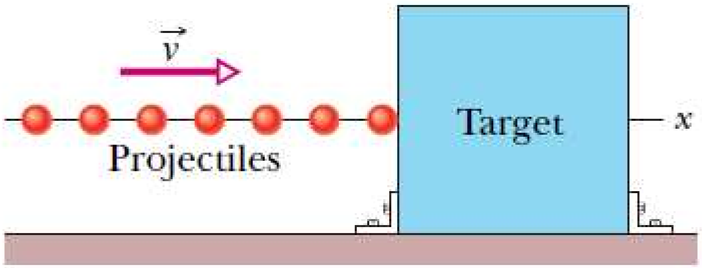


Fig. 9-10 A steady stream of projectiles, with identical linear momenta, collides with a target, which is fixed in place. The average force F_{avg} on the target is to the right and has a magnitude that depends on the rate at which the projectiles collide with the target or, equivalently, the rate at which mass collides with the target.

Let n be the number of projectiles that collide in a time interval Δt.

Each projectile has initial momentum mv and undergoes a change Δp in linear momentum because of the collision.

The total change in linear momentum for n projectiles during interval Δt is nΔp. The resulting impulse on the target during Δt is along the x axis and has the same magnitude of nΔp but is in the opposite direction.

$$J = -n\,\Delta p,$$

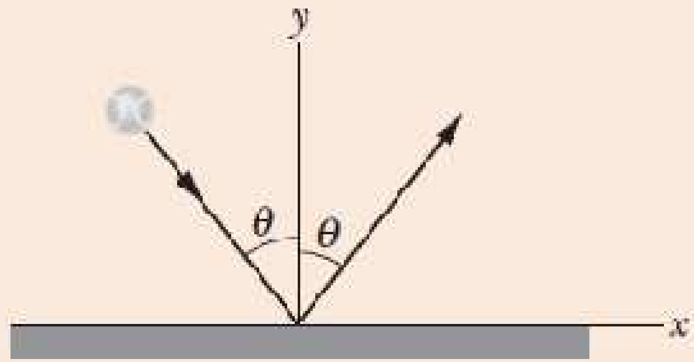
$$F_{avg} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m\,\Delta v.$$

In time interval Δt, an amount of mass Δm =nm collides with the target.

$$F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v.$$

CHECKPOINT 5

The figure shows an overhead view of a ball bouncing from a vertical wall without any change in its speed. Consider the change $\Delta\vec{p}$ in the ball's linear momentum. (a) Is Δp_x positive, negative, or zero? (b) Is Δp_y positive, negative, or zero? (c) What is the direction of $\Delta\vec{p}$?



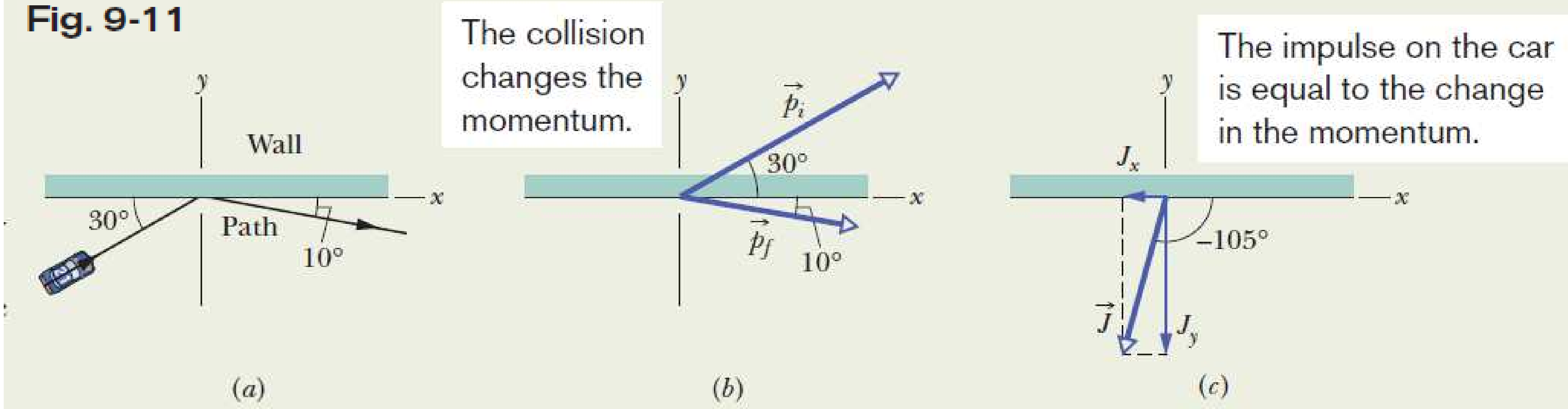
Sample problem: 2-D impulse

Race car–wall collision. Figure 9-11*a* is an overhead view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed $v_i = 70$ m/s along a straight line at 30° from the wall. Just after the collision, he is traveling at speed $v_f = 50$ m/s along a straight line at 10° from the wall. His mass m is 80 kg.

(a) What is the impulse $\vec{J}$ on the driver due to the collision?

(b) The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?

Fig. 9-11



9.7: Conservation of Linear Momentum

If no net external force acts on a system of particles, the total linear momentum, **P**, of the system cannot change.

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system).}$$

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

$$\vec{P}_i = \vec{P}_f \qquad \text{(closed, isolated system).}$$

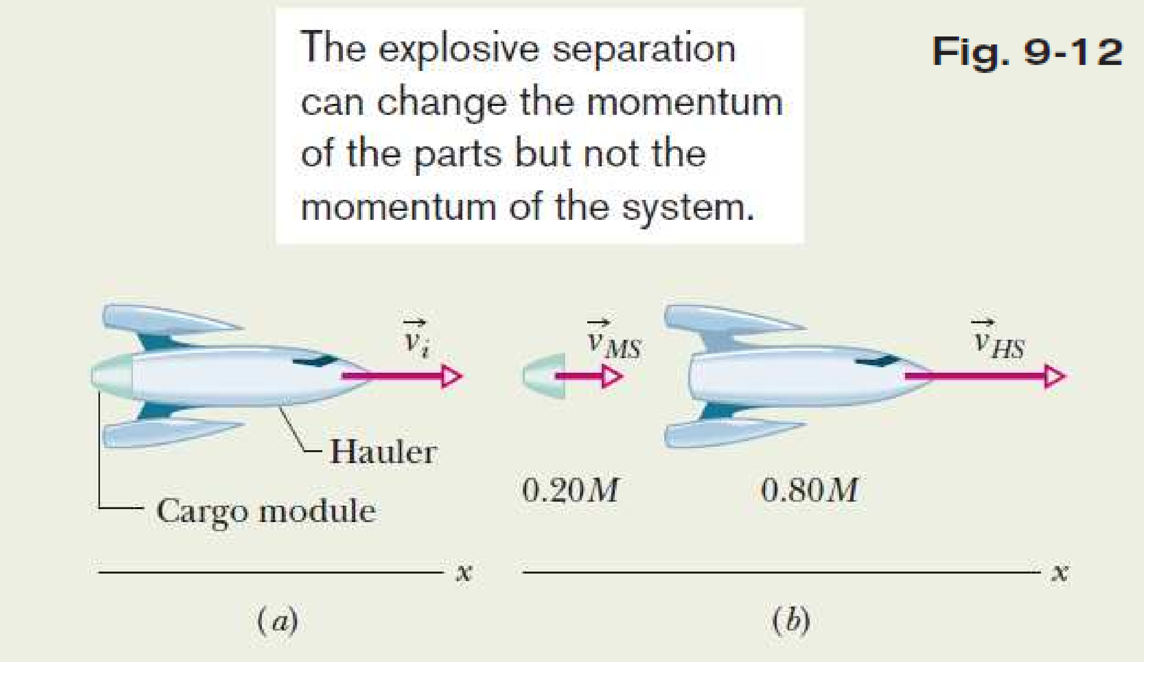
$$\begin{pmatrix} \text{total linear momentum} \\ \text{at some initial time } t_i \end{pmatrix} = \begin{pmatrix} \text{total linear momentum} \\ \text{at some later time } t_f \end{pmatrix}.$$

CHECKPOINT 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor. One piece slides in the positive direction of an x axis. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the x axis? (c) What is the direction of the momentum of the second piece?

Sample problem: 1-D explosion

One-dimensional explosion: Figure 9-12*a* shows a space hauler and cargo module, of total mass M, traveling along an x axis in deep space. They have an initial velocity $\vec{v}_i$ of magnitude 2100 km/h relative to the Sun. With a small explosion, the hauler ejects the cargo module, of mass $0.20M$ (Fig. 9-12*b*). The hauler then travels 500 km/h faster than the module along the x axis; that is, the relative speed v_{rel} between the hauler and the module is 500 km/h. What then is the velocity $\vec{v}_{HS}$ of the hauler relative to the Sun?



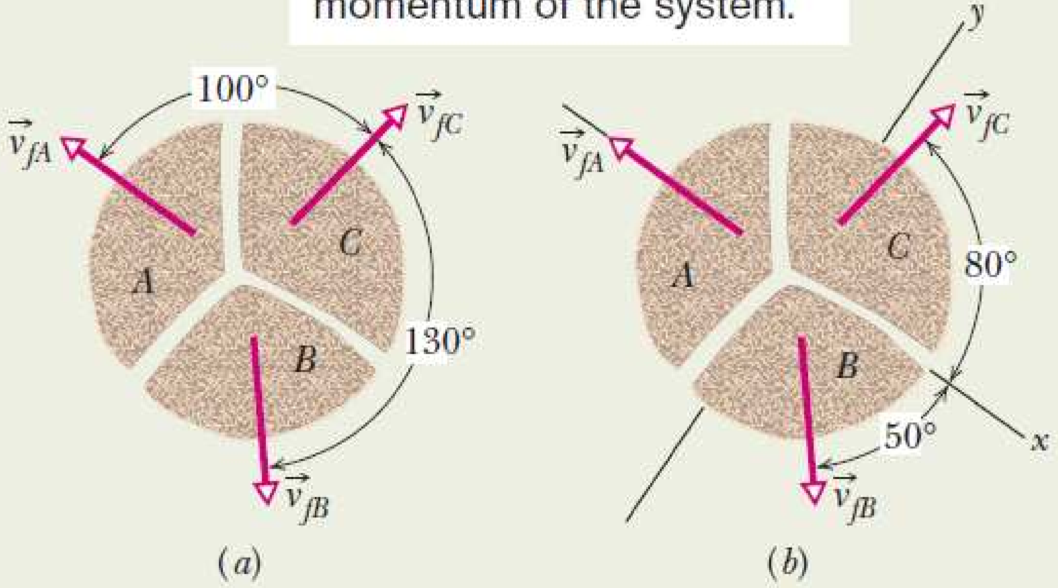
Sample problem: 2-D explosion

Two-dimensional explosion: A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown in Fig. 9-13*a*. Piece C, with mass $0.30M$, has final speed $v_{fC} = 5.0$ m/s.

(a) What is the speed of piece B, with mass $0.20M$?

(b) What is the speed of piece A?

Fig. 9-13 The explosive separation can change the momentum of the parts but not the momentum of the system.



9.8: Momentum and Kinetic Energy in Collisions

In a closed and isolated system, if there are two colliding bodies, and the total kinetic energy is unchanged by the collision, then the kinetic energy of the system is conserved (it is the same before and after the collision). Such a collision is called an ***elastic collision*.**

If during the collision, some energy is always transferred from kinetic energy to other forms of energy, such as thermal energy or energy of sound, then the kinetic energy of the system is not conserved. Such a collision is called an ***inelastic collision*.**

9.9: Inelastic collisions in 1-D

Here is the generic setup for an inelastic collision.

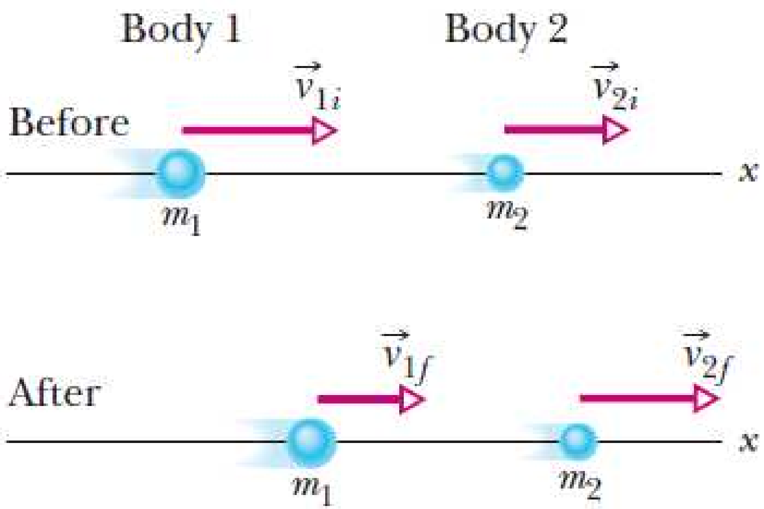


Fig. 9-14 Bodies 1 and 2 move along an x axis, before and after they have an inelastic collision.

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}.$$

In a completely inelastic collision, the bodies stick together.

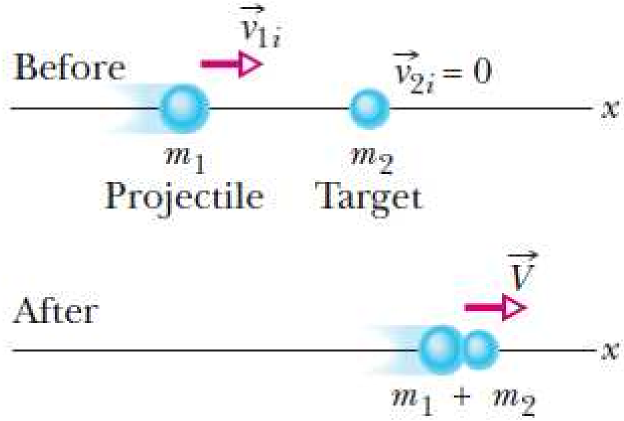


Fig. 9-15 A completely inelastic collision between two bodies. Before the collision, the body with mass m_2 is at rest and the body with mass m_1 moves directly toward it. After the collision, the stuck-together bodies move with the same velocity $\vec{V}$.

$$m_1v_{1i} = (m_1 + m_2)V$$

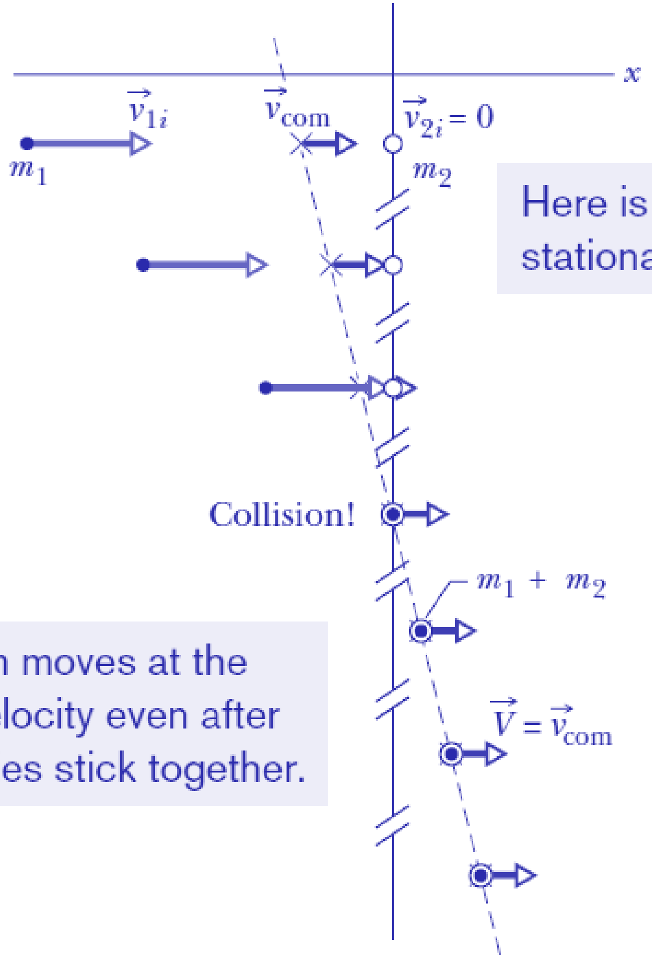
$$V = \frac{m_1}{m_1 + m_2}v_{1i}.$$

9.9: Inelastic collisions in 1-D: Velocity of Center of Mass

The com of the two bodies is between them and moves at a constant velocity.

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$

Here is the incoming projectile.



Here is the stationary target.

The com moves at the same velocity even after the bodies stick together.

Fig. 9-16 Some freeze frames of a two-body system, which undergoes a completely inelastic collision. The system's center of mass is shown in each freeze-frame.

The velocity v_{com} of the center of mass is unaffected by the collision. Because the bodies stick together after the collision, their common velocity V must be equal to v_{com}.

CHECKPOINT 7

Body 1 and body 2 are in a completely inelastic one-dimensional collision. What is their final momentum if their initial momenta are, respectively, (a) 10 kg · m/s and 0; (b) 10 kg · m/s and 4 kg · m/s; (c) 10 kg · m/s and −4 kg · m/s?

Sample problem: conservation of momentum

The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown in Fig. 9-17 consists of a large block of wood of mass $M = 5.4$ kg, hanging from two long cords. A bullet of mass $m = 9.5$ g is fired into the block, coming quickly to rest. The *block + bullet* then swing upward, their center of mass rising a vertical distance $h = 6.3$ cm before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?

There are two events here. The bullet collides with the block. Then the bullet–block system swings upward by height *h*.

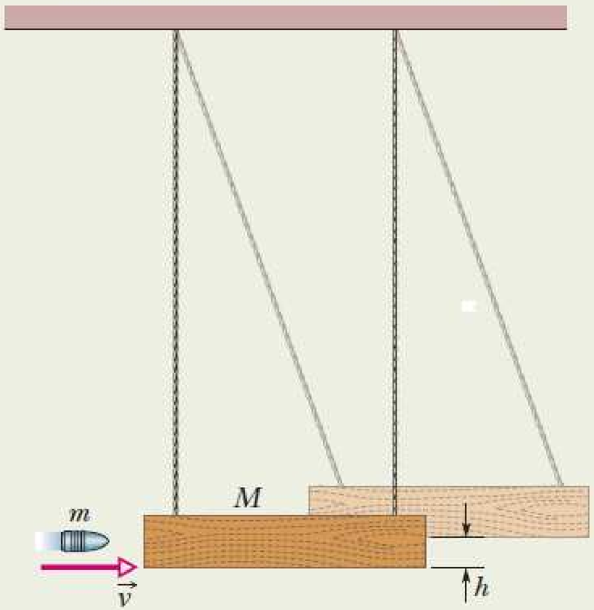


Fig. 9-17 A ballistic pendulum, used to measure the speeds of bullets.

9.10: Elastic collisions in 1-D

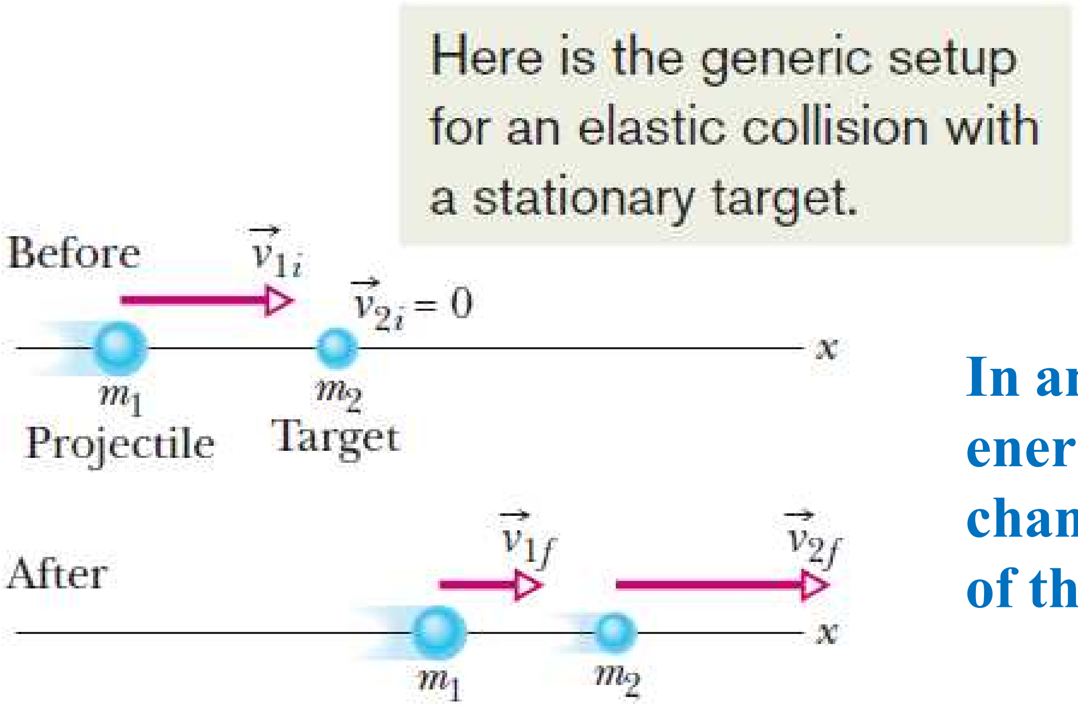


Fig. 9-18 Body 1 moves along an x axis before having an elastic collision with body 2, which is initially at rest. Both bodies move along that axis after the collision.

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

9.10: Elastic collisions in 1-D: Stationary Target

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{(linear momentum).}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad \text{(kinetic energy).}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Here is the generic setup for an elastic collision with a stationary target.

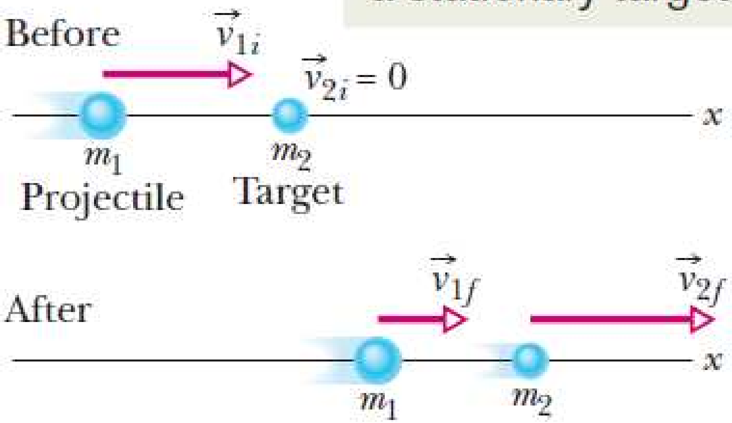


Fig. 9-18 Body 1 moves along an x axis before having an elastic collision with body 2, which is initially at rest. Both bodies move along that axis after the collision.

9.10: Elastic collisions in 1-D: Moving Target

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f},$$

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2.$$

Here is the generic setup for an elastic collision with a moving target.

Fig. 9-19 Two bodies headed for a one-dimensional elastic collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$

CHECKPOINT 8

What is the final linear momentum of the target in Fig. 9-18 if the initial linear momentum of the projectile is 6 kg · m/s and the final linear momentum of the projectile is (a) 2 kg · m/s and (b) −2 kg · m/s? (c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are, respectively, 5 J and 2 J?

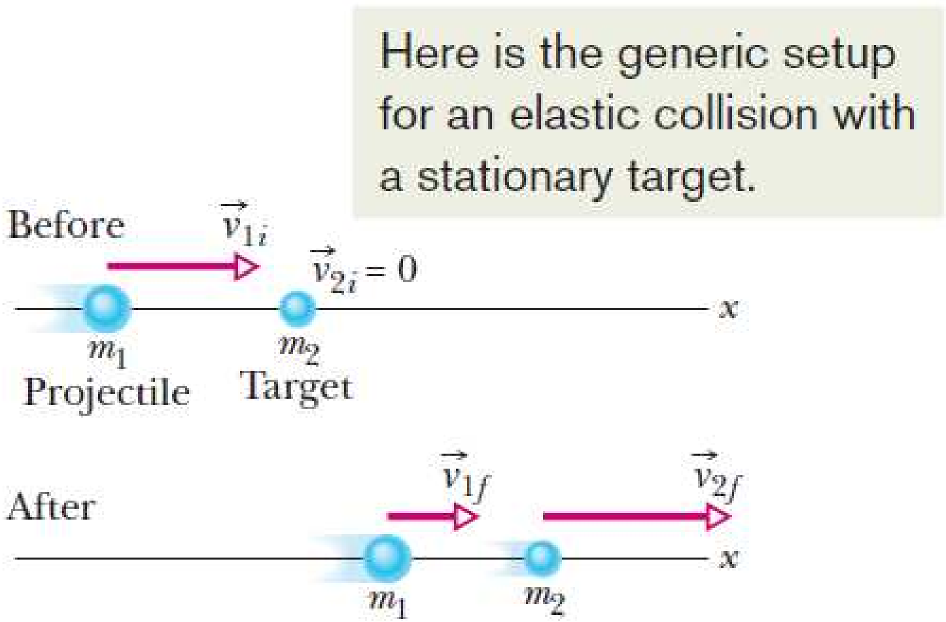
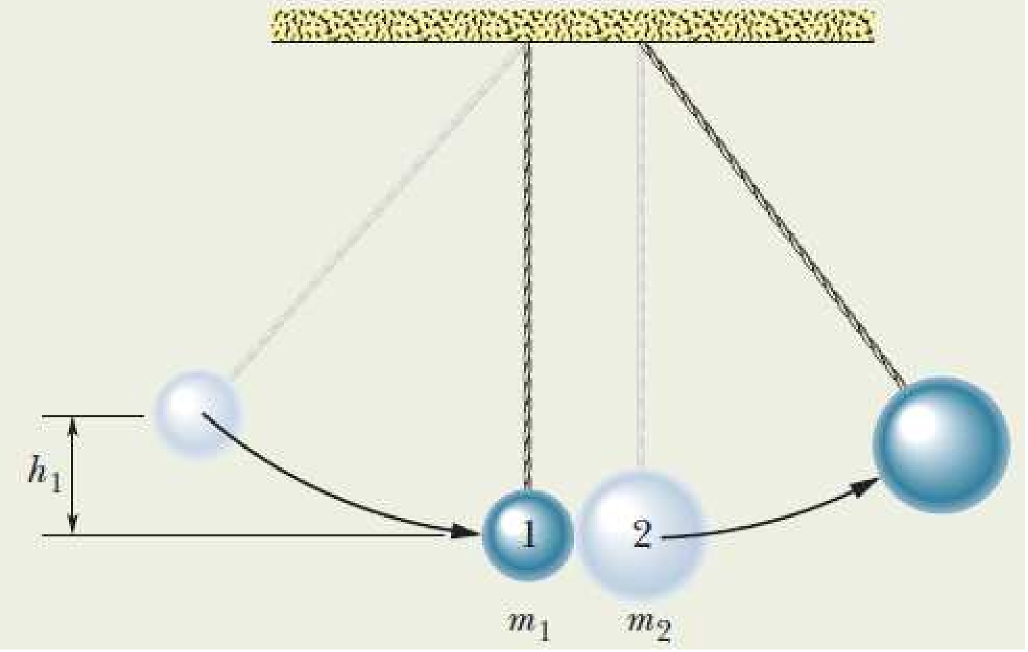


Fig. 9-18 Body 1 moves along an x axis before having an elastic collision with body 2, which is initially at rest. Both bodies move along that axis after the collision.

Sample problem: two pendulums

Two metal spheres, suspended by vertical cords, initially just touch, as shown in Fig. 9-20. Sphere 1, with mass $m_1 = 30$ g, is pulled to the left to height $h_1 = 8.0$ cm, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass $m_2 = 75$ g. What is the velocity v_{1f} of sphere 1 just after the collision?



9.11: Collisions in 2-D

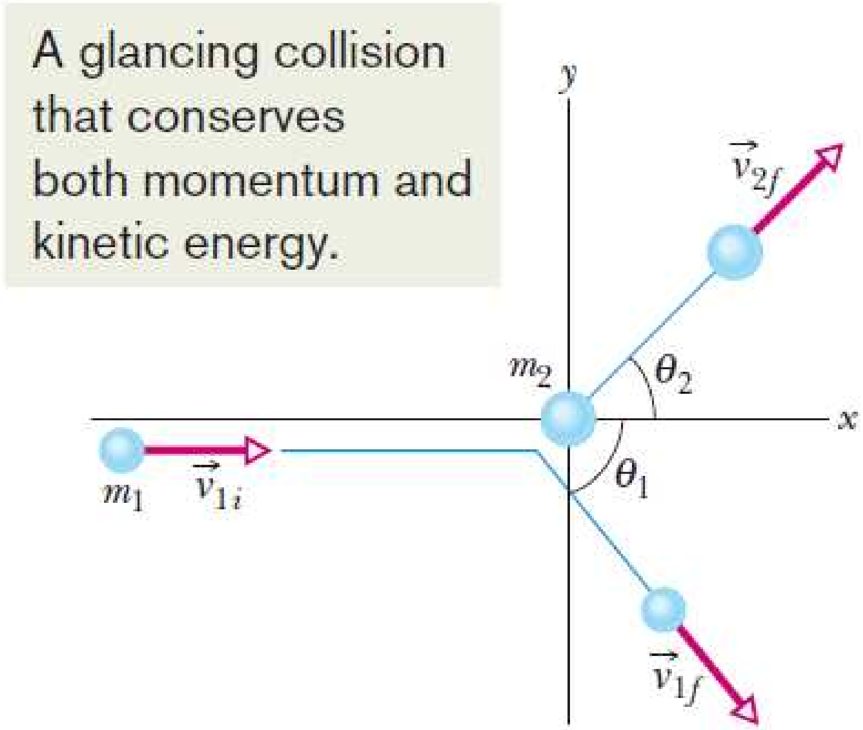


Fig. 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}.$$

If elastic,

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}.$$

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2,$$

$$0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2.$$

Also,

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

CHECKPOINT 9

In Fig. 9-21, suppose that the projectile has an initial momentum of 6 kg · m/s, a final x component of momentum of 4 kg · m/s, and a final y component of momentum of -3 kg · m/s. For the target, what then are (a) the final x component of momentum and (b) the final y component of momentum?

A glancing collision that conserves both momentum and kinetic energy.

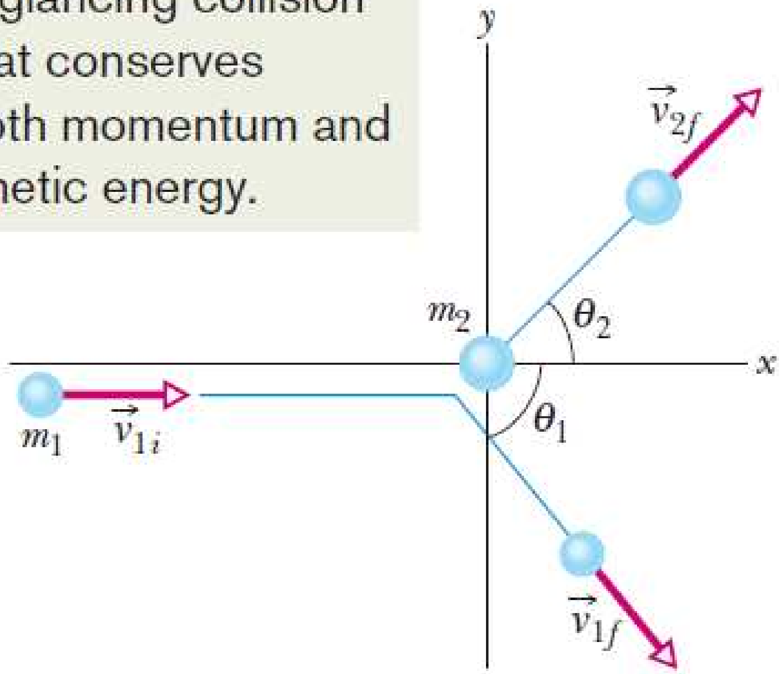


Fig. 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

9.12: Systems with Varying Mass: A Rocket

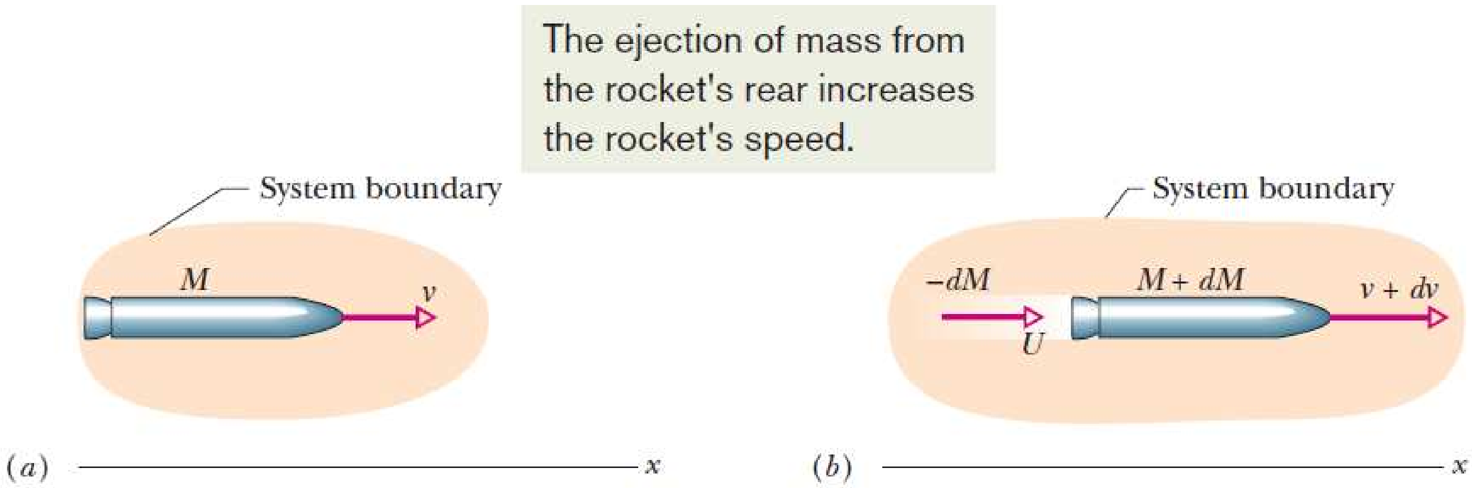


Fig. 9-22 (*a*) An accelerating rocket of mass M at time t, as seen from an inertial reference frame. (*b*) The same but at time $t + dt$. The exhaust products released during interval dt are shown.

The system here consists of the rocket and the exhaust products released during interval dt. The system is closed and isolated, so the linear momentum of the system must be conserved during dt, where the subscripts i and f indicate the values at the beginning and end of time interval dt.

$$P_i = P_f, \quad \Longrightarrow \quad Mv = -dM\,U + (M + dM)(v + dv)$$

$$\begin{pmatrix}\text{velocity of rocket}\\ \text{relative to frame}\end{pmatrix} = \begin{pmatrix}\text{velocity of rocket}\\ \text{relative to products}\end{pmatrix} + \begin{pmatrix}\text{velocity of products}\\ \text{relative to frame}\end{pmatrix}$$

$$(v + dv) = v_{rel} + U,$$
$$U = v + dv - v_{rel}.$$

$$\Longrightarrow \quad -\frac{dM}{dt}v_{rel} = M\frac{dv}{dt}. \quad \Longrightarrow \quad Rv_{rel} = Ma$$

9.12: Systems with Varying Mass: Finding the velocity

$$dv = -v_{\text{rel}} \frac{dM}{M}.$$

$$\int_{v_i}^{v_f} dv = -v_{\text{rel}} \int_{M_i}^{M_f} \frac{dM}{M},$$

in which M_i is the initial mass of the rocket and M_f its final mass. Evaluating the integrals then gives

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f}$$

for the increase in the speed of the rocket during the change in mass from M_i to M_f .

Sample problem: rocket engine, thrust, acceleration

A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

KEY IDEA

Thrust T is equal to the product of the fuel consumption rate R and the relative speed v_{rel} at which exhaust gases are expelled, as given by Eq. 9-87.

Calculation: Here we find

$$T = Rv_{rel} = (2.3\text{ kg/s})(2800\text{ m/s})$$
$$= 6440\text{ N} \approx 6400\text{ N}. \qquad \text{(Answer)}$$

(b) What is the initial acceleration of the rocket?

KEY IDEA

We can relate the thrust T of a rocket to the magnitude a of the resulting acceleration with $T = Ma$, where M is the rocket's mass. However, M decreases and a increases as fuel is consumed. Because we want the initial value of a here, we must use the intial value M_i of the mass.

Calculation: We find

$$a = \frac{T}{M_i} = \frac{6440\text{ N}}{850\text{ kg}} = 7.6\text{ m/s}^2. \qquad \text{(Answer)}$$

To be launched from Earth's surface, a rocket must have an initial acceleration greater than $g = 9.8\text{ m/s}^2$. That is, it must be greater than the gravitational acceleration at the surface. Put another way, the thrust T of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude $M_i g$, which gives us

$$(850\text{ kg})(9.8\text{ m/s}^2) = 8330\text{ N}.$$

Because the acceleration or thrust requirement is not met (here $T = 6400$ N), our rocket could not be launched from Earth's surface by itself; it would require another, more powerful, rocket.